Write your name below. This exam is worth 100 points. On each problem (except for problem 1), you must show all your work to receive credit on that problem. You are NOT allowed to use your notes, book, calculator, or any other electronic devices.

Name:

1. (21 points: 3 each) If the statement is **always true** mark "TRUE"; if it is possible for the statement to be false then mark "FALSE". **No justification is necessary**.

   (a) Let B be the square matrix obtained by exchanging two rows of the square matrix A and let $det(A) < det(B)$ then A is nonsingular.

   (b) In an inner-product space if $\|f\| = \|g\|$, where $\|\cdot\|$ is the norm defined from the inner product, then $f \equiv g$.

   (c) If all entries of a 5×5 matrix A are 5, then $det(A) = 5^5$.

   (d) If A and B are symmetric invertible matrices, then ABA^{-1} is also symmetric and invertible.

   (e) If f and g are elements in an inner product space satisfying $\|f\| = 2, \|g\| = 4$ and $\|f+g\| = 5$, then it is possible to find the exact value of $\langle f, g\rangle$.

   (f) If $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions then $A\mathbf{x} = \mathbf{0}$ has an infinite number number of solutions as well.

   (g) Let $v, w \in \mathbb{R}^3$ it holds that $\|w\| \leq \|v\| + \|w+v\|$, where $\|\cdot\|$ is any norm in $\mathbb{R}^3$.

2. (19 points) Consider the following matrix A

$$A = \begin{bmatrix} 0 & 3 & 0 & 2 \\ 1 & 2 & -3 & 0 \\ 2 & 5 & -4 & 3 \\ -3 & -4 & 7 & 0 \end{bmatrix}.$$

   (a) (7 points) Find the permutation matrix P such that $B := PA$ is symmetric. Show both P and B.

   (b) (12 points) Can B be factored as LDL^T? If yes, find the factorization. If no, justify why it cannot be factored.

3. (20 points: 10 each)

   The following two problems are unrelated.

   (a) Determine if the following matrices are linearly independent

   $$\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}, \begin{pmatrix} 3 & -1 \\ 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & -5 \\ -4 & 0 \end{pmatrix}.$$

   (b) Let $V = \mathbb{R}^4$ and $W \subset V$ be the space spanned by the vectors: $\begin{pmatrix} 1 \\ -2 \\ 5 \\ -3 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \\ -4 \end{pmatrix}, \begin{pmatrix} 3 \\ 8 \\ -3 \\ -5 \end{pmatrix}.$

   Find a basis and dimension for W.

4. (19 points)

   The following two questions are unrelated.

   (a) (9 points) Let $V = \mathbb{R}^3$ and $W = \left\{(x, y, z)^T \in V : \; x^2 - 2xy + y^2 - z^2 = 0\right\}$. Is W a vector subspace of V? Prove or disprove.

   (b) (10 points) Consider $\mathcal{F}(I)$, the vector space of real valued functions on an interval I. Do the solutions to the differential equation

   $$y'' + 5y' + 2y = 0$$

   form a subspace of $\mathcal{F}(I)$? Prove that they do or show that they do not.

5. (21 points)

   Let $A = \begin{pmatrix} 1 & 2 & 1 & 2 & 0 \\ 2 & 4 & 3 & 4 & 1 \\ 1 & 2 & 2 & 2 & 1 \end{pmatrix}$.

   (a) (3 points) What is the rank of A?

   (b) (3 points) What is $\dim \operatorname{coker} A$?

   (c) (5 points) Find a basis for the image of A.

   (d) (5 points) Find a basis for the coimage of A.

   (e) (5 points) Find a basis for the kernel of A.