Write your name and your professor's name or your section number in the top right corner of your paper. You are allowed to use textbooks and notes, but you may not ask anyone for help except the proctors. To receive full credit on a problem you must show **sufficient justification for your conclusion** unless explicitly stated otherwise.

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1. (30 points: 3 each) If the statement is **always true** mark "TRUE"; if it is possible for the statement to be false then mark "FALSE." **No justification is necessary.**

_____(a) If $L(\boldsymbol{x}) = \mathbf{A}\mathbf{x}$ is a linear function from $\mathbf{R}^n$ to $\mathbf{R}^n$, and the columns of $\mathbf{B}$ form a basis of $\mathbb{R}^n$, then $\mathbf{B}\mathbf{A}\mathbf{B}^{-1}$ is the matrix representation of L with respect to the new basis.

_____(b) If $\|\cdot\|$ is a norm on vector space V, then the formula $\frac{1}{4}\left(\|\boldsymbol{u}+\boldsymbol{v}\|^2 - \|\boldsymbol{u}-\boldsymbol{v}\|^2\right)$ defines an inner product $\langle \boldsymbol{u}, \boldsymbol{v}\rangle$ on V.

_____(c) The 1-norm of an $n \times n$ matrix $\mathbf{A}$ is $\max_i \sum_{j=1}^n |a_{ij}|$.

_____(d) Suppose $\mathbf{A}$ has full column rank, and QR factorization $\mathbf{QR}$ (the Gram-Schmidt version). True or False: $\mathbf{R}^T$ is the Cholesky factor of the Gram matrix $\mathbf{A}^T\mathbf{A}$.

_____(e) The determinant of a Householder elementary reflection matrix is -1.

_____(f) If c_i are the coordinates of $\boldsymbol{b}$ with respect to an orthonormal basis, then $\|\boldsymbol{b}\|^2 = \sum_i c_i^2$ (the norm is the one associated with the inner product).

_____(g) Let V be a finite-dimensional inner product space and let W be a subspace of V. True or False: $\left(W^\perp\right)^\perp = W$.

_____(h) The orthogonal projection of $\boldsymbol{b}$ onto the range of $\mathbf{A}$ (using the dot product) is given by $\mathbf{A}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\boldsymbol{b}$.

_____(i) The orthogonal complement of the kernel (also called the null space) of $\mathbf{A}$ is the corange (also called the row space or co-image) of $\mathbf{A}$. (Orthogonality is defined using the dot product.)

_____(j) If $\boldsymbol{b}$ is orthogonal to cokernel (also called the left-hand null space) of $\mathbf{A}$, then $\mathbf{A}\boldsymbol{x} = \boldsymbol{b}$ has a solution.

2. (15 points) Suppose that $\mathbf{A}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\2\\0\end{pmatrix}$ and $\mathbf{A}\begin{pmatrix}1\\-1\\2\end{pmatrix} = \begin{pmatrix}0\\1\\3\end{pmatrix}$. Solve $\mathbf{A}\mathbf{x} = \begin{pmatrix}1\\0\\6\end{pmatrix}$.

3. (20 points) Prove that the following expression defines an inner product:

$$\langle \boldsymbol{u}, \boldsymbol{v} \rangle = 2u_1(v_1 - 2v_2) + u_2(-4v_1 + 9v_2 + v_3) + u_3(v_2 + 4v_3)$$

4. (20 points) Find the QR factorization of $\mathbf{A} = \begin{bmatrix} 1 & -2 & -1 \\ 1 & -2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ using the Gram-Schmidt version of QR.

5. (15 points) Find the critical point of this function, and explain whether it is a maximizer, minimizer, or neither:

$$q(x, y, z) = 2x^2 - 8xy + 9y^2 + 2yz + 4z^2 + 4x - 12y - 10z - \sqrt{581}$$