Write your name and your professor's name or your section number in the top right corner of your paper. You are allowed to use textbooks and notes, but you may not ask anyone for help except the proctors. To receive full credit on a problem you must show **sufficient justification for your conclusion** unless explicitly stated otherwise.

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1. (30 points: 3 each) If the statement is **always true** mark "TRUE"; if it is possible for the statement to be false then mark "FALSE." **No justification is necessary.**

_____(a) If $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are all square invertible matrices of the same size, then $(\mathbf{B}\mathbf{C}^T\mathbf{A}^{-1})^{-1} = \mathbf{B}^{-1}(\mathbf{C}^T)^{-1}\mathbf{A}$.

_____(b) If $\mathbf{A}$ is a square invertible (aka nonsingular) matrix, then so is $\mathbf{A}^n$ for any positive integer n.

_____(c) Let $\vec{x} = (1,2)^T$ be a column vector and $\vec{y} = (-1,2)$ be a row vector. True or false: $\det(\vec{x}\vec{y}) = 3$.

_____(d) If $\mathbf{A}\vec{x} = \vec{b}$ has two solutions, then it has infinitely many solutions.

_____(e) If an upper-triangular matrix is invertible, then its inverse is lower triangular.

_____(f) If $\mathbf{A}$ is a square matrix with $\det(\mathbf{A}) = 0$, then $\mathbf{A}\vec{x} = \vec{b}$ has no solutions.

_____(g) If the rank of $\mathbf{A}$ is less than the number of columns of $\mathbf{A}$, then the linear system of equations $\mathbf{A}\vec{x} = \vec{b}$ has an infinite number of solutions.

_____(h) Let V be the set of 3×4 matrices with the usual definitions of addition and scalar multiplication. This is a vector space. Consider the subset W of matrices with rank 1. True or False: W is a subspace of V.

_____(i) If $\mathbf{A}$ is 3×4 with rank 3, then the columns of $\mathbf{A}$ span $\mathbb{R}^3$.

_____(j) If $\mathbf{A}$ is 4×6 with rank 2, then the kernel of $\mathbf{A}$ is two dimensional.

2. Let $A = \begin{bmatrix} 0 & 2 & 2 \\ 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$.

(a) (8 points) Find the permuted LU factorization of $\mathbf{A}$.

(b) (12 points) Use the permuted LU factorization to solve $\mathbf{A}\vec{x} = \begin{pmatrix} 0 \\ -2 \\ 2 \end{pmatrix}$.

(c) (5 points) Use the permuted LU factorization to find the determinant of $\mathbf{A}$.

3. (10 points) Let $V = \mathbb{R}^3$ and let W be the subset consisting of vectors $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ such that $x^2 + 2xy + y^2 = 0$. Prove that W is a subspace, or find an example showing that it is not closed under addition or scalar multiplication.

4. (15 points) Do the following functions span the vector space of polynomials of degree ≤ 2?

$$\left\{1, 1 - x, 1 + 2x - x^2, x^3\right\}$$

5. Let $\mathbf{B} = \begin{bmatrix} 1 & 0 & 3 \\ -1 & 1 & -1 \\ 2 & -3 & 0 \end{bmatrix}$. The row echelon form of $\mathbf{B}$ is $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$.

   (a) (4 points) Find a basis for the range (aka image) of $\mathbf{B}$.

   (b) (4 points) Find a basis for the corange (aka coimage) of $\mathbf{B}$.

   (c) (4 points) Find a basis for the kernel of $\mathbf{B}$.

   (d) (8 points) Find a basis for the cokernel of $\mathbf{B}$.