Write your name below. You must show your work and not give decimal answers (i.e. don't use a calculator or software to compute a decimal answer). You are not allowed to collaborate on the exam or seek outside help, though using your notes, the book, the recorded lectures, or material you find online is acceptable (you can't ask someone for help online). To receive full credit on a problem you must show **sufficient justification for your conclusion** unless explicitly stated otherwise. Please submit this exam to the course canvas page by **December 13 at 11:59PM (Mountain Time)**.

Name:

1. (20 points: 2 each) If the statement is **always true** mark "TRUE" and provide a *brief* justification; if it is possible for the statement to be false then mark "FALSE" and provide a counterexample or the correct statement.

   (a) If $\mathbf{A}$ is a 4×2 matrix and $\mathbf{B}$ is a 3×4 matrix, then the product $\mathbf{BA}$ is defined.

   (b) $\mathbf{A}$ is nonsingular if and only if $\mathbf{A}$ has an eigenvalue 0. (You may assume that $\mathbf{A}$ is square.)

   (c) If $\mathbf{A} = \mathbf{TDT}^{-1}$ for some diagonal matrix $\mathbf{D}$ and invertible matrix $\mathbf{T}$, then the columns of $\mathbf{T}$ are eigenvectors of $\mathbf{A}$.

(d) Let $\mathbf{A} = \mathbf{P}\boldsymbol{\Sigma}\boldsymbol{Q}^T$ be the SVD of $\mathbf{A}$. True or false: The columns of $\mathbf{P}$ are eigenvectors of $\mathbf{A}\mathbf{A}^T$.

(e) If $\mathbf{A}$ has full column rank and $\mathbf{A}^\dagger$ is the pseudoinverse of $\mathbf{A}$, then $\mathbf{A}\mathbf{A}^\dagger$ is an orthogonal projection matrix that projects onto the range of $\mathbf{A}$.

(f) If $\mathbf{A}$ is an $m \times n$ matrix and the linear system $\mathbf{A}\mathbf{x} = \mathbf{b}$ has two free variables, then $\operatorname{rank}(A) = n$.

(g) If λ is an eigenvalue of matrix $\mathbf{A}$, then λ^2 is an eigenvalue of $\mathbf{A}^2$.

(h) Suppose $\mathbf{A}$ is a symmetric matrix. Then, $\mathbf{A}\mathbf{x} = \mathbf{b}$ has a solution if and only if $\mathbf{b}$ is orthogonal to the corange of $\mathbf{A}$.

(i) If $\mathbf{A}$ is invertible, then $\mathbf{A}$ is diagonalizable.

(j) The singular values of a nonsingular matrix $\mathbf{A}$ are the same as the singular values of $\mathbf{A}^{-1}$.

2. Consider the following linear transformation: $L(x, y) = \begin{pmatrix} -x + y \\ 2x - y \end{pmatrix}$.

   (a) (4 points) Find the matrix form of $L(x, y)$ with respect to the standard basis.

   (b) (6 points) Find the matrix form of $L(x, y)$ with respect to the following basis: $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$, $\vec{v}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. (The same basis is used for both the domain space and the co-domain space.)

3. (20 points) Find the Jordan decomposition of $\mathbf{A} = \begin{bmatrix} -3 & 3 & -3 \\ -1 & -1 & -1 \\ -2 & 1 & -2 \end{bmatrix}$.

4. (20 points) Find the singular value decomposition of

$$\mathbf{A} = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}.$$

5. (10 points) Find the LU factorization of the following matrix. Do not use row permutations.

$$\begin{bmatrix} 2 & 1 & 0 \\ -4 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

6. (10 points) Find bases for both the range and corange of the following matrix. (Be sure your answer is supported by your work.)

$$\mathbf{A} = \begin{bmatrix} 1 & -3 & -3 \\ -2 & 4 & 2 \\ -1 & 5 & 7 \end{bmatrix}.$$

7. (10 points) Find the least-squares solution of the system

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & 1 & -1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 5 \\ 6 \\ 6 \end{bmatrix}.$$