Write your name and your professor's name or below. You must show your work and not give decimal answers (i.e. don't use a calculator or software to compute a decimal answer). You are not allowed to collaborate on the exam or seek outside help, though using your notes, the book, the recorded lectures, or material you find online is acceptable (you can't ask someone for help online). To receive full credit on a problem you must show **sufficient justification for your conclusion** unless explicitly stated otherwise. Please submit this exam to the course canvas page by **October 21 at 11:59PM (Mountain Time)**.

Name:

Instructor/Section:

1. (30 points: 5 each) If the statement is **always true** mark "TRUE" and provide a *brief* justification; if it is possible for the statement to be false then mark "FALSE" and provide a counterexample.

   (a) Suppose that W is a nonempty subset of another set V. In other words, $W \subset V$ and W is not the empty set, $\{\}$. Then W is a *subspace* of V if the following two features are true:

   (i) for every $\mathbf{v}, \mathbf{w} \in W$, the sum $\mathbf{v} + \mathbf{w} \in W$, and

   (ii) for every $\mathbf{v} \in W$ and every $c \in \mathbb{R}$, the scalar product $c\mathbf{v} \in W$.

   (b) The vectors $\mathbf{u} = (1, 2, -1)^T$, $\mathbf{v} = (3, 1, 5)^T$, and $\mathbf{w} = (8, 1, 2)^T$ are linearly dependent.

   (c) The columns of the following matrix span $\mathbb{R}^3$:

$$\mathbf{A} = \begin{bmatrix} 3 & 4 & -1 \\ -2 & 7 & 3 \\ 3 & 2 & -1/2 \end{bmatrix}.$$

(d) If $\mathbf{A}$ is an $m \times n$ matrix and the linear system $\mathbf{Ax} = \mathbf{b}$ has no free variables, then $\text{rank}(A) = n$.

(e) Let $\mathbf{A}$ be the following 3×3 matrix:

$$\begin{bmatrix} 1 & 0 & -1 \\ 4 & 3 & 2 \\ 2 & 1 & 3 \end{bmatrix}.$$

Then $||\mathbf{A}||_\infty = 7$.

(f) It is always true that $||\mathbf{A} + \mathbf{B} + \mathbf{C}|| \leq ||\mathbf{A}|| + ||\mathbf{B}|| + ||\mathbf{C}||$.

2. Let $\mathbf{A} = \begin{bmatrix} 4 & 0 & 5 \\ 2 & 3 & -1 \\ -2 & 1 & 0 \end{bmatrix}$.

   (a) (10 points) Find a basis for the *kernel* of the columns of $\mathbf{A}$.

   (b) (10 points) Find a basis for the *range* of $\mathbf{A}$.

3. Let $\mathbf{A} = \begin{bmatrix} 0 & 1 & -3 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$.

   (a) (10 points) What is the Frobenius norm of $\mathbf{A}$?

   (b) (10 points) What is the 1-norm of $\mathbf{A}$?

4. Consider the matrix $\mathbf{A} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}$

   (a) (10 points) Is $\mathbf{A}$ *symmetric positive definite*?

   (b) (10 points) Is $\mathbf{A}^{\mathbf{T}}$ *symmetric positive definite*?

5. (10 points) Find the angle between the following vectors using the standard dot product:

$$\begin{bmatrix} 2 & -1 & 0 \end{bmatrix}^T, \text{ and } \begin{bmatrix} -1 & 2 & -1 \end{bmatrix}^T.$$