Write your name and your professor's name below. You must show your work and not give decimal answers (i.e. don't use a calculator or software to compute a decimal answer). You are not allowed to collaborate on the exam or seek outside help, though using your notes, the book, the recorded lectures, or material you find online is acceptable (you can't ask someone for help online). To receive full credit on a problem you must show **sufficient justification for your conclusion** unless explicitly stated otherwise. Please submit this exam to the course canvas page by **September 23 at 11:59PM (Mountain Time)**.

Name:

Instructor:

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1. (30 points: 5 each) If the statement is **always true** mark "TRUE" and provide a *brief* justification; if it is possible for the statement to be false then mark "FALSE" and provide a counterexample.

   (a) A linear system of equations consisting of the same number of variables and equations always has a unique solution.

   (b) The determinant of a nonsingular matrix is 0.

   (c) If $\mathbf{A}$ is a 4×3 matrix and $\mathbf{B}$ is a 2×4 matrix, then the product $\mathbf{BA}$ is defined.

(d) If a 2×2 upper-triangular matrix is invertible, then its inverse is also upper-triangular.

(e) If $\mathbf{Ax} = \mathbf{b}$ is incompatible, then so is $\mathbf{Ax}{=}\mathbf{c}$.

(f) If A is invertible, then the determinant of $\mathbf{A}$ is the same as the determinant of $\mathbf{A}^{-1}$.

2. Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 4 \\ 3 & 0 & 1 \end{bmatrix}$.

   (a) (10 points) Use Gauss-Jordan Elimination to find the inverse of $\mathbf{A}$.

   (b) (10 points) Use your answer from (a) to find the solution of

$$\begin{cases} x_1 & & = -3 \\ & x_2 + 4x_3 & = 1 \\ 3x_1 & + \quad x_3 & = 0 \end{cases}$$

3. Let $\mathbf{A} = \begin{bmatrix} 0 & 1 & -3 \\ 0 & 2 & 3 \\ 1 & 0 & 2 \end{bmatrix}$.

   (a) (10 points) Determine the permuted LU-factorization of $\mathbf{A}$.

   (b) (10 points) What is the determinant of $\mathbf{A}$?

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(c) (10 points) Use the permuted LU-factorization from (a) to solve the system $\mathbf{A}\boldsymbol{x} = (1, 5, 2)^T$. You must use the permuted LU factorization to receive points. Other methods will receive no points.

4. (10 points) Suppose that $n \times n$ matrices $\mathbf{A}$ and $\mathbf{B}$ are similar. That is, there exists an invertible matrix $\mathbf{S}$ such that $\mathbf{B} = \mathbf{S}^{-1}\mathbf{A}\mathbf{S}$. Prove that $\mathbf{A}$ and $\mathbf{B}$ have the same determinant.

5. (10 points) Find all solutions of the following system:

$$\begin{cases} x_1 + 4x_2 - 2x_3 = -3 \\ 2x_1 + x_2 + 3x_3 = 1 \end{cases}$$