- This exam is worth 100 points and has 4 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/071423 (20 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The equation of motion for a particular harmonic oscillator is $x(t) = \cos 2t - \sin 2t$. An equivalent expression is $x(t) = \sqrt{2}\cos\left(2t - \frac{\pi}{4}\right)$.

   (b) Any finite order, linear, homogeneous, constant coefficient differential equation that does not contain an undifferentiated term will always have a constant solution.

   (c) The differential equation $x'' + e^x = 2$ describes a conservative system.

   (d) The function $x(t) = e^{2t} + e^{-3t}$ is a possible solution to the differential equation for a harmonic oscillator governed by $mx'' + bx' + kx = 0$ with mass m, damping constant b and restoring constant k.

   (e) The solution space of the differential equation $y''' = 0$ is span $\left\{t^2 + t, t^2 + 2, 4t - 8\right\}$.

2. [2360/071423 (27 pts)] Let $\mathrm{L}(\vec{\mathbf{y}}) = 2t^2 y'' + 5ty' + y$.

   (a) (8 pts) Find a basis for the solution space of $\mathrm{L}(\vec{\mathbf{y}}) = 0$ by assuming solutions of the form $y = t^r$.

   (b) (4 pts) Verify that your basis functions are indeed solutions to the equation.

   (c) (15 pts) Find the general solution of $\mathrm{L}(\vec{\mathbf{y}}) = 2t^4$ using variation of parameters.

3. [2360/071423 (20 pts)] Characteristic equations for certain constant coefficient linear homogeneous differential equations are given, along with a forcing function, $f(t)$. Give the form of the particular solution you would use to solve the nonhomogeneous differential equations from which the characteristic equations were derived when using the Method of Undetermined Coefficients. **Do not** solve for the coefficients.

   (a) $r(r-2)(r-1) = 0;\ f(t) = 2 + \sin t$

   (b) $[r - (-2-2i)][r - (-2+2i)] = 0;\ f(t) = \cos 2t + t$

   (c) $(r+4)(r-2) = 0;\ f(t) = e^{2t} + e^{4t}$

   (d) $[r - (1-i)][r - (1+i)](r+3)^2 = 0;\ f(t) = e^t \cos t + e^{-t}\sin t + te^{-3t}$

   (e) $r^3(r-1) = 0;\ f(t) = \cos 2t - \sin 3t + 1$

4. [2360/071423 (33 pts)] A 2-kg mass is attached to spring with restoring/spring constant of 2 Nt/m. The apparatus is aligned horizontally with a damping constant of 5 Nt/m/sec, and is forced by $f(t) = 3e^{-t} + 4$ Nt. Initially, $x(0) = -4$ and $\dot{x}(0) = -3$.

   (a) (2 pts) Where is the mass with respect to its equilibrium position when $t = 0$ and in what direction is it moving at that time?

   (b) (3 pts) Is the oscillator over-, under-, or critically damped? Justify your answer.

   (c) (3 pts) Is the oscillator in resonance? Justify your answer.

   (d) (15 pts) Find the position of the mass at any time t, that is, solve an appropriate initial value problem.

   (e) (2 pts) From your answer to part (d), identify the transient and steady state solutions.

   (f) (8 pts) Write the initial value problem from part (d) as a system of differential equations/IVPs, using matrices and vectors in your answer.