- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/063023 (24 pts)] The following parts (a) and (b) are not related.

   (a) (12 pts) Suppose that $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ are $n \times n$ matrices with $|\mathbf{A}| = 0$, $|\mathbf{B}| = 3$, $|\mathbf{C}| = 1$ and that $\mathbf{D}$ is an $m \times n$ matrix. Calculate the following or explain why they fail to exist.

      i. $|\mathbf{CB}|$
      ii. $|\mathbf{C}^{\mathrm{T}}|$
      iii. $\left|\mathbf{C}^2 \left(\mathbf{A}^{\mathrm{T}}\right)^{-1}\right|$
      iv. $|\mathbf{DA}|$

   (b) (12 pts) Let $\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 1 & 3 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 2 \end{bmatrix}$. Calculate the following or explain why they fail to exist.

      i. $\mathbf{AB}$
      ii. $\mathbf{AB}^{\mathrm{T}}$
      iii. $\mathbf{A} + \mathbf{B}$
      iv. $\left(\mathbf{A}^{\mathrm{T}}\mathbf{A}\right)^{\mathrm{T}}$

2. [2360/063023 (16 pts)] The following parts (a) and (b) are not related.

   (a) (12 pts) Decide if the following subsets, $\mathbb{W}$, of the given vector space, $\mathbb{V}$, are subspaces. Assume the standard operations of vector addition and scalar multiplication apply. Justify the correct answer completely for full credit. A simple yes/no will result in zero points.

      i. $\mathbb{V} = \mathcal{C}([0,1])$; $\mathbb{W} = \left\{ f(t) \in \mathcal{C}([0,1]) \;\middle|\; \int_0^1 f(t)\,\mathrm{d}t = 0 \right\}$
      ii. $\mathbb{V} = \mathbb{M}_{22}$; $\mathbb{W} = \left\{ \begin{bmatrix} 1 & a \\ b & 1 \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$
      iii. $\mathbb{V} = \mathbb{R}^3$; $\mathbb{W} = \left\{ \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} \;\middle|\; a, b \in \mathbb{R} \right\}$

   (b) (4 pts) Determine whether or not the set $\mathbb{S} = \{4, t - t^2, t^3\}$ forms a basis for some vector space. If so, what is the vector space's dimension?

3. [2360/063023 (19 pts)] Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$.

   (a) (6 pts) Show that $\lambda = 0$ and $\lambda = 2$ are eigenvalues of the matrix.
   (b) (4 pts) State the algebraic multiplicity for the eigenvalues in part (a).
   (c) (6 pts) Find the eigenspace associated with $\lambda = 0$ and state its dimension.
   (d) (3 pts) Is it possible that the system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$, where $\vec{\mathbf{b}} \neq \vec{\mathbf{0}}$, could be inconsistent? Explain briefly.

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4. [2360/063023 (21 pts)] The following problems are not related.

(a) (6 pts) Find the RREF of the matrix $\mathbf{A} = \begin{bmatrix} 3 & 0 & 6 \\ 1 & 3 & 2 \\ 2 & 6 & 4 \end{bmatrix}$.

(b) (15 pts) The following is the augmented matrix from a system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$.

$$\left[\begin{array}{ccccc|c} 0 & 1 & 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

i. (3 pts) Is the matrix in RREF? If so, write YES. If not, put the matrix in RREF.

ii. (6 pts) Find a particular solution to the nonhomogeneous system.

iii. (6 pts) Find a basis for the solution space of the associated homogeneous problem. What is the dimension of this solution space?

5. [2360/063023 (20 pts)] Let $\vec{\mathbf{v}} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $\vec{\mathbf{b}} = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$.

(a) (5 pts) Compute $\vec{\mathbf{v}}^{\mathrm{T}}\vec{\mathbf{v}}$ and $\mathrm{Tr}\left(\vec{\mathbf{v}}^{\mathrm{T}}\vec{\mathbf{v}}\right)$.

(b) (5 pts) Can Cramer's Rule be used to solve the linear system $\left(\vec{\mathbf{v}}\vec{\mathbf{v}}^{\mathrm{T}}\right)\vec{\mathbf{x}} = \vec{\mathbf{b}}$? Justify your answer.

(c) (10 pts) Compute $\mathbf{H} = \mathbf{I} - 2\vec{\mathbf{v}}\vec{\mathbf{v}}^{\mathrm{T}}$ and use its inverse to solve $\mathbf{H}\vec{\mathbf{x}} = \vec{\mathbf{b}}$. Perform Gauss-Jordan elimination to find the inverse.