- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting.

---

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/061623 (18 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The operator $\mathrm{L}(y) = [\ln(\sin t)]y' + 3y$ is a linear operator.

   (b) Let $y' = f(y)$, where f is continuous. An $f(y)$ exists such that there are only two equilibrium solutions, both of which are semistable.

   (c) The equation $(y')^2 + (\sin t)y + 1 = 0$ is linear, second order, homogeneous, autonomous differential equation.

   (d) The differential equation $(\cos t)y' + (\sin t)y = t$ can be solved using integrating factor method.

   (e) Picard's theorem guarantees the existence of a unique solution to the initial problem: $y' = (\ln t)y^{1/2}$, $y(1) = 0$.

   (f) The differential equation describing Newton's Law of Cooling possesses a single, stable equilibrium solution.

2. [2360/061623 (20 pts)] The following questions are unrelated.

   (a) (14 pts) Solve the following initial value problem

   $$\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{e^{-y}}{2t-1},\ y(0) = 0.$$

   (b) (6 pts) Consider the following initial value problem

   $$y' = (\ln t)y,\ y(e) = 1.$$

   Apply two steps of Euler's method using a step size $h = e$.

3. [2360/061623 (20 pts)] Consider the following system of differential equations:

   $$\begin{aligned} x' &= x^2 + y^2 - 4 \\ y' &= y - xy \end{aligned}$$

   (a) (5 pts) Find the h nullclines

   (b) (5 pts) Find the v nullclines

   (c) (6 pts) On a single phase plane, plot the h nullclines as solid curves/lines and the v nullclines as dashed curves/lines.

   (d) (4 pts) Find all equilibrium points, provided any exist.

4. [2360/061623 (18 pts)] A tank with a total capacity of 4600 liters (L) initially contains 600 L of fresh water. Water containing $\dfrac{\ln(t+1)}{(t+1)^2}$ kilograms (kg) of sugar per liter flows into the tank at a rate of 6 L/min, and the well mixed sugar solution is allowed to drain from the bottom of the tank at a rate of 2 L/min. Let $x(t)$ denote the amount (kg) of sugar in the tank at time t.

   (a) (14 pts) Write down, but **do not solve**, the initial value problem that $x(t)$ satisfies.

   (b) (4 pts) Without solving the initial value problem from part (a), state the interval over which the solution will be valid based on the physical situation.

CONTINUED ON REVERSE

5. [2360/061623 (24 pts)] Consider the differential equation $ty' = y + \dfrac{1}{t}$.

(a) (4 pts) Show that $y_h = Ct$ is the general solution of the associated homogeneous equation, noting that C is an arbitrary constant.

(b) (15 pts) Use variation of parameters (Euler-Lagrange two-stage method) to find a particular solution to the original nonhomogeneous equation. Show all your work.

(c) (5 pts) Find the solution to the differential equation passing through the point $\left(2, \dfrac{3}{4}\right)$.