- This exam is worth 100 points and has 5 questions.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on one side.

At the top of the page containing your answer to problem 1, write the following italicized statement and sign your name to it: "*I will abide by the CU Boulder Honor Code on this exam.*" **FAILURE TO INCLUDE THIS MAY RESULT IN A PENALTY.**

1. [2360/061022 (24 pts)] A 200 liter tank initially contains 100 liters (L) of pure water. Water enters the tank at a rate of 2 L/hr and the water entering the tank has a kool-aid concentration of 2 gram/L. If a well-mixed solution leaves the tank at a rate of 1 L/hr, how much kool-aid powder is dissolved in the tank when it overflows? Be sure to define the variables you use and use the integrating factor method.

2. [2360/061022 (16 pts)] Find the general solution of $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 + \sqrt{y - 2x + 3}$ by using the substitution $u = y - 2x + 3$. Write your solution as an explicit function (be sure to simplify).

3. [2360/061022 (18 pts)] The following parts are not related.

   (a) (6 pts) Consider the system of differential equations

   $$\begin{aligned} x' &= y^4 + 1 \\ y' &= x + 1 \end{aligned}$$

   i. Find the h and v nullclines of the system.

   ii. Find the equilibrium points, if any exist.

   (b) (12 pts) Consider the initial value problem $(x + 1)w'(x) + w = 2(x + 1)$, $w(1) = 5$, $x \geq 0$. The solution to the associated homogeneous equation is $w_h(x) = c(x + 1)^{-1}$.

   i. Complete the Euler-Lagrange two stage method (variation of parameters) to find the particular solution.

   ii. Solve the initial value problem.

4. [2360/061022 (16 pts)] The following parts are not related.

   (a) (8 pts) Consider the differential equation $y' = (y-1)^{2/3}e^{2t}\tan 2t$. What conclusions, if any, can be drawn from Picard's theorem regarding the existence and uniqueness of solutions to the initial value problems consisting of the differential equation and the following initial conditions:

   i. $y(0) = 1$ ii. $y(\pi/2) = 0$

   (b) (8 pts) Use one step of Euler's method to approximate the solution of $y' = 2\pi t \sin y$, $y(1) = \pi/2$ at $t = 1.5$.

5. [2360/061022 (26 pts)] The following parts are not related.

   (a) (16 pts) Consider the differential equation $x'(t) = x^2(x^2 + 1)(x^2 + x - 6)$.

   i. (6 pts) Find all the equilibrium solutions and determine their stability.

   ii. (7 pts) Draw the phase line of the equation.

   iii. (3 pts) Let $x_1(t)$ be the solution that passes through the point $(0, -2)$. What is $\lim\limits_{t\to\infty} x_1(t)$?

   (b) (10 pts) On your paper, write the letters A-E in a column. Next to each letter, write the order of each differential equation followed by the numbers (I-III) corresponding to all the classes to which the differential equation belongs. No justification required and no partial credit available. (I) separable (II) linear (III) homogeneous

   A. $y' - 1 = 0$ B. $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \sqrt{1 + \left(\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)^2}$ C. $y''' + 6y'' - 8y = \sin t$

   D. $(\sin t)x'' = x'$ E. $e^y y' - x\tan x = 0$