1. [APPM 2360 Exam (20 pts)] Consider the nonhomogeneous differential equation $t^2y'' - 3ty' + 3y = t^4,\ t > 0$.
   (a) Assuming solutions of the form $y = t^r$, solve the associated homogeneous equation.
   (b) Show that your solutions from part (a) form a basis for the solution space of the homogeneous equation.
   (c) Find a particular solution to the nonhomogeneous equation.
   (d) Solve the initial value problem consisting of the nonhomogeneous differential equation along with the initial conditions $y(1) = -\frac{2}{3}, y'(1) = \frac{7}{3}$.

2. [APPM 2360 Exam (20 pts)] Find the general solution of $\frac{\mathrm{d}^4y}{\mathrm{d}t^4} - 4\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = 12t - 16$. Use the Method of Undetermined Coefficients to find a particular solution.

3. [APPM 2360 Exam (20 pts)] Consider the linear operator $L(\vec{\mathbf{y}}) = y''' - 6y'' + 13y' - 10y$.
   (a) (7 pts) If $y = e^{2t}$ is one solution to the equation $L(\vec{\mathbf{y}}) = 0$, find the general solution of $L(\vec{\mathbf{y}}) = 0$.
   (b) (8 pts) Now consider $L(\vec{\mathbf{y}}) = f(t)$. Write down the form of the particular solution y_p to use in the Method of Undetermined Coefficients for the given $f(t)$. Do not find the constants.
      i. $f(t) = 5e^t + e^{2t} - 1$.
      ii. $f(t) = te^{2t}$
      iii. $f(t) = \cos 5t + \sin 7t$
      iv. $f(t) = 10e^{2t}\sin t$
   (c) (5 pts) Convert the initial value problem $L(\vec{\mathbf{y}}) = e^{-t} + 6,\ y(0) = 4, y'(0) = 3, y''(0) = 7$ into a system of three first order differential equations, writing your answer in the form $\vec{\mathbf{x}}' = \mathbf{A}\vec{\mathbf{x}} + \vec{\mathbf{f}}(t)$. Be sure to include the initial condition in your answer.

4. [APPM 2360 (20 pts)] The following problems are not related.
   (a) Consider an harmonic oscillator governed by the differential equation $m\ddot{x} + b\dot{x} + x = A\cos\left(\frac{1}{4}t\right)$.
      i. (3 pts) Find the values of A, m and b so that the oscillator will exhibit resonance.
      ii. (3 pts) Find the values of A, m and b so that the equation will have bounded solutions.
      iii. (2 pts) If the mass of the oscillator is 1 unit, find the values of A and b such that the oscillator will be unforced and the mass will pass through the equilibrium position as most once.
   (b) (12 pts) Now consider an undamped oscillator with mass 1 unit, restoring/spring constant 1 unit that starts from rest at the equilibrium position. It is oriented horizontally and is driven by the function $f(t) = \sin t$. After $3\pi/2$ units of time have elapsed:
      i. In relation to the equilibrium position, where is the mass?
      ii. How fast and in what direction is the mass moving?

5. [APPM 2360 Exam (20 pts)] A mass of 1 kg is attached to a spring whose constant is 5 N/m. Initially, the mass is released 1 m below the equilibrium position with a downward velocity of 5 m/s, and the subsequent motion takes place in a medium that offers a damping force that is numerically equal to 2 times the instantaneous velocity.
   (a) (15 pts) Find the equation of motion if the mass is driven by an external force equal to $f(t) = 12\cos 2t + 3\sin 2t$.
   (b) (5 pts) Find the amplitude of the steady-state solution.