1. [APPM 2360 Exam (20 pts)] Let $\vec{\mathbf{u}} = \begin{bmatrix} \sqrt{2}/2 \\ \sqrt{2}/2 \\ 0 \end{bmatrix}$

   (a) (6 pts) Show that $\vec{\mathbf{u}}^{\mathrm{T}}\vec{\mathbf{u}} = \mathbf{I}_1$, where $\mathbf{I}_1$ is the 1×1 identity matrix.

   (b) (6 pts) Compute $\mathbf{H} = \mathbf{I}_3 - 2\vec{\mathbf{u}}\,\vec{\mathbf{u}}^{\mathrm{T}}$

   (c) (6 pts) Show that $\mathbf{H}$ is nonsingular (invertible).

   (d) (2 pts) Can Cramer's Rule be used to solve the system $\left(\vec{\mathbf{u}}\,\vec{\mathbf{u}}^{\mathrm{T}}\right)\vec{\mathbf{x}} = \vec{\mathbf{b}}$? Explain why or why not.

2. [APPM 2360 Exam (15 pts)] Consider the linear system

$$\begin{aligned} x - \ \ 4y &= \ \ 17 \\ 3x - 12y &= \ \ k \\ -2x + \ \ 8y &= -34 \end{aligned}$$

   (a) (8 pts) Use Gauss-Jordan Reduction to determine the value of k that makes the system consistent.

   (b) (7 pts) Using the value of k found in part (a), write the solution to the system using the Nonhomogenous Principle $\vec{\mathbf{x}} = \vec{\mathbf{x}}_h + \vec{\mathbf{x}}_p$.

3. [APPM 2360 Exam (20 pts)] Use the matrix inverse to find the solution to the system

$$\begin{aligned} x_1 \ + 2x_2 + 3x_3 &= 3 \\ x_2 + 2x_3 &= 1 \\ -2x_1 \qquad\ + 3x_3 &= 4 \end{aligned}$$

4. [APPM 2360 (25 pts)] The following problems are not related.

   (a) (12 pts) Suppose $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are $n \times n$ matrices with $|\mathbf{A}| = 3, |\mathbf{B}| = 1, |\mathbf{C}| = 0$. Calculate the following or explain why they fail to exist.

      i. $|\mathbf{AB}|$
      ii. $\left|\mathbf{B}^{\mathrm{T}}\right|$
      iii. $\left|\mathbf{B}^2\mathbf{A}\mathbf{C}^{-1}\right|$
      iv. $|\mathbf{D}|$, where $\mathbf{D}$ is the matrix obtained by interchanging the second and n^{th} rows of $\mathbf{A}$

   (b) (9 pts) Let $\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 3 & 2 \\ -1 & 0 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 & -2 \\ 1 & 1 \\ 0 & 3 \end{bmatrix}$. Calculate the following or explain why they fail to exist.

      i. $\mathbf{A}\mathbf{B}^{\mathrm{T}}$
      ii. $\mathbf{A}^{\mathrm{T}} + \mathbf{B}$
      iii. $\left(\mathbf{B}^{\mathrm{T}}\right)^{-1}$

   (c) (4 pts) Let $\mathbf{A}$ be an $n \times n$ invertible matrix satisfying $\mathbf{A}^3 + 2\mathbf{A} = \mathbf{I}$. Find an expression for $\mathbf{A}^{-1}$.

5. [APPM 2360 Exam (20 pts)] The following problems are not related.

   (a) (8 pts) Decide if the following subsets $\mathbb{W}$ of the given vector space $\mathbb{V}$ are subspaces. Assume that the standard operations of vector addition and scalar multiplication apply. Justify the correct answer completely for full credit. A simple yes/no will result in zero points.

      i. $\mathbb{V} = C\left([0,1]\right); \quad \mathbb{W} = \left\{ f(t) \,\middle|\, \int_0^1 f(t)\,\mathrm{d}t = 2 \right\}$
      ii. $\mathbb{V} = \mathbb{M}_{23}; \quad \mathbb{W} =$ matrices of the form $\begin{bmatrix} 0 & a & b \\ c & 0 & d \end{bmatrix}$ where a, b, c, d are real numbers.

   (b) (4 pts) Determine whether or not the set $S = \left\{2, 1-t, t+t^3\right\}$ forms a basis for some vector space. If so, what is its dimension? If not, explain why not.

   (c) (8 pts) Find the eigenvalues and eigenvectors of $\mathbf{A} = \begin{bmatrix} 2 & -12 \\ 1 & -5 \end{bmatrix}$.