On the front of your Bluebook write: (1) your name, (2) your section number: **300** for Thaler or **301** for Sprenger and (3) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one sided crib sheet is allowed, which you can take with you when you finish the exam. Make sure to read all instructions carefully and box your final answer.

1. (12 pts) **True/False** (answer True if it is always true otherwise answer False). No justification is required as there is no partial credit on this question.
   (a) The differential equation $\ddot{x} - 2\dot{x} + 4x = 0$ describes a critically damped harmonic oscillator.
   (b) The set $\{e^{2t}, e^{-t}, te^{-t}\}$ forms a basis for the solution space of $y''' - 3y' - 2y = 0$.
   (c) A second order differential equation always has two linearly independent solutions.
   (d) If y_1 and y_2 are solutions to $ay'' + by' + cy = f(t)$, then $y = y_1 - y_2$ is a solution to $ay'' + by' + cy = 0$.

2. (20 pts) The following questions are unrelated. Answer each question and justify your response for full credit.
   (a) Determine the basis for the solution space of the differential equation $y^{(4)} - y = 0$.
   (b) For which values of ω_f does the system $\ddot{x} + 4x = \cos(\omega_f t)$ exhibit resonance?
   (c) Determine the steady state solution to the forced mass spring system

   $$\ddot{x} + 2\dot{x} + 3x = 4\cos(t)$$

   (d) For the following, determine the form of the particular solution so that we can apply the method of undetermined coefficients. If the method of undetermined coefficients can not be used, clearly state so.
      i. $y'' + 5y = e^{-t}$
      ii. $y'' - 4y' - 12y = t^{-2}$
      iii. $y'' + y = t\cos t$

3. (24 pts) An object with mass 2 kg is attached to a spring with spring constant $k = 1$ N/m. The unforced motion is damped with a damping constant of 2 N/m/s. The position of the mass at any time t will be denoted by $x(t)$.
   (a) Write a differential equation for the position of the mass, $x(t)$.
   (b) Solve the differential equation you found in part (a). You must show your work to receive full credit.
   (c) If the initial displacement of the mass is $x(0) = 2$ and the initial velocity is $x'(0) = 0$, what is the motion of the mass, $x(t)$, corresponding to this initial value problem?
   (d) Sketch the solution determined in part (c).
   (e) Characterize the motion of the oscillator as underdamped, overdamped, or critically damped.

There are more problems on the back!

4. (20 points) In this problem we will investigate the initial value problem

$$\begin{aligned} y'' - y &= 8te^t \\ y(0) &= 1 \\ y'(0) &= 1 \end{aligned}$$

(a) Find the homogeneous solution of the differential equation.

(b) Determine an appropriate guess for the form of the particular solution.

(c) Use the method of undetermined coefficients to solve for the particular solution using your guess in part (b).

(d) Determine the solution of the initial value problem.

5. (24 points) Consider the differential equation

$$y''' + 2y'' + y' = e^t$$

(a) Use the variable substitution $x = y'$ (so $x' = y''$ and $x'' = y'''$) to write a second order differential equation for x.

(b) Solve for the homogeneous solution, x_h, of the differential equation you found in part (a).

(c) Use the method of *variation of parameters* to determine the particular solution, x_p, to the equation you found in part (a).

(d) Write the general solution to the differential equation in part (a).

(e) Find $y(t)$, the solution of the original differential equation, using the formula $y(t) = \displaystyle\int x(t)\,\mathrm{d}t + C$, where C is a constant.