APPM 2360 **Exam 2** **Summer 2019**

On the front of your Bluebook write: (1) your name, (2) your section number: **300** for Thaler or **301** for Sprenger and (3) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one sided crib sheet is allowed, which you can take with you when you finish the exam. Make sure to read all instructions carefully and $\boxed{\text{box}}$ your final answer.

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1. (15 pts) **True/False** (answer True if it is always true otherwise answer False). No justification is required as there is no partial credit on this question.

   (a) The vector $\vec{\mathbf{v}} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ is an eigenvector of the matrix $\begin{bmatrix} 2 & 2 \\ 5 & -1 \end{bmatrix}$.

   (b) The dimension of the vector space $\mathbb{P}_3$ is 3.

   (c) if P and D are invertible matrices, then $(PDP^{-1})^{-1} = PD^{-1}P^{-1}$.

   (d) If B is an $n \times n$ matrix with nonzero determinant, then span$\{$col $B\} = \mathbb{R}^n$, where col B denotes the column space of B.

   (e) The set of all triangular (upper and lower) 2×2 matrices is a vector subspace of $\mathbb{M}_{2\times 2}$.

2. (20 pts) The following questions are unrelated. Answer each question and justify your response for full credit.

   (a) Find the dimension of the subspace of $\mathbb{P}_3$ spanned by

   $$\left\{t^3 + t^2,\ t^3 - 2t - 1,\ t^2 + t,\ t + 1\right\}.$$

   (b) Determine the basis for the space of matrices $A = \begin{bmatrix} 2a & b \\ 3a + b & b \end{bmatrix}$, where $a, b \in \mathbb{R}$.

   (c) Compute A^{-1} for $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$.

   (d) For the given matrix $A = \begin{bmatrix} 0 & 4 \\ 1 & 2 \end{bmatrix}$, compute the eigenvalues λ_1 and λ_2 and show that $\det(A) = \lambda_1\lambda_2$.

3. (25 pts) For the following problem, we consider the linear system of equations

   $$\begin{aligned} x - 2y + \ \ z &= \ \ 0 \\ 2x + \ \ y - 3z &= \ \ 5 \\ 4x - 7y + \ \ z &= -1 \end{aligned}$$

   (a) Rewrite the system as an augmented matrix $(A|\vec{\mathbf{b}})$ where A is a matrix and $\vec{\mathbf{b}}$ is a column vector.

   (b) Use Gauss-Jordan elimination to write the matrix from part (a) in reduced row echelon form (RREF).

   (c) Based on your result in part (b), what is the solution to linear system of equations? Verify this result by plugging in your result to the original system.

   (d) Determine the solution to the problem $A\vec{\mathbf{x}} = \mathbf{0}$, where A is the same matrix as in part (a).

There are more problems on the back!

4. (20 pts) In this problem we will consider the matrix

$$A = \begin{bmatrix} 1 & 0 & 10 \\ 0 & 7+k & -3 \\ 0 & 4 & k \end{bmatrix}$$

(a) For which values of k is A invertible?

(b) Compute the eigenvalues of A when $k = -2$.

(c) Using the eigenvalues computed in (b), compute the eigenvector(s) corresponding to the *smallest* eigenvalue of A when $k = -2$.

5. (20 pts) In the following problems, $\mathbb{W}$ is a subset of a vector space $\mathbb{V}$. Determine whether $\mathbb{W}$ is a subspace. If $\mathbb{W}$ is a subspace, prove it. Otherwise provide an explanation or counterexample to show why $\mathbb{W}$ does not form a subspace of $\mathbb{V}$. No credit will be given for responses without justification.

(a) $\mathbb{V} = \mathbb{M}_{2\times 2}$, $\mathbb{W}$ is the set of matrices $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ such that $a + d = 0$.

(b) $\mathbb{V} = \mathcal{C}([0,1])$, $\mathbb{W} = \left\{ f(x) \in \mathcal{C}([0,1]) \;\middle|\; \int_0^1 f(x)dx = 0 \right\}$.

(c) $\mathbb{V} = \mathcal{C}^1(-\infty, \infty)$, $\mathbb{W} = \left\{ f(x) \,|\, f' = f^2 \right\}$.