APPM 2360: Midterm Exam 3
July 13, 2018

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your recitation section number and (4) a grading table. Textbooks, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **1 side only**) note sheet is allowed.

Problem 1: (40 points) Answer each part of this question with TRUE or FALSE. **DO NOT** write T or F. Also, **provide a brief ONE SENTENCE justification.**

(a) The harmonic oscillator modeled by the differential equation

$$2\ddot{x} + 5\dot{x} + 3x = e^{-3t}$$

will not experience resonance.

(b) A basis for the solution space for the differential equation

$$y'' + 2y' + y = 0$$

has three linearly independent functions.

(c) For the forced oscillator $m\ddot{x}+b\dot{x}+kx = F\cos(\omega_f t)$, beats occur when the forcing frequency, ω_f, is equal to the natural frequency, $\omega_0 = \sqrt{k/m}$.

(d) The method of undetermined coefficients can be used to find a particular solution to $t^2y'' + 3y = \cos t$

Problem 2: (40 points) The following problems are unrelated.

(a) Is $\{e^{-2t}, e^t\}$ a basis for the solution space of $\ddot{x} + 5\dot{x} + 6x = 0$? Be sure to show your work and justify your answer. Don't just say "Yes" or "No".

(b) For a mass-spring system $m\ddot{x}+b\dot{x}+kx = 0$, state the three types of damped motion and, in each case, provide the equation in terms of $m, b,$ and k for when that type of motion occurs. In each of the three cases, your answer should be in the form "_____-damped motion occurs when _____" where you fill in the blank spaces.

(c) Find the general solution for the following homogeneous linear differential equation

$$y^{(4)} + 4y''' + 3y'' = 0$$

Problem 3: (40 points)
Consider the differential equation

$$x'' - 4x' + 9x = f(t)$$

(a) Find the homogeneous solution for this differential equation.

(b) Given the following forcing functions, provide the initial guess for the form of the particular solutions for undetermined coefficients. **DO NOT** solve for the particular solution.
   (i) $f(t) = t^2 + 3$
   (ii) $f(t) = e^{2t}\sin\left(\sqrt{5}t\right)$
   (iii) $f(t) = t^2e^t$

(c) Use **variation of parameters** to solve for the particular solution when $f(t) = e^{2t}$.

Problem 4: (40 points) There are "runner's highs" and "runner's lows" when you feel elated and terrible, respectively, on a long run. Suppose your mood on a long run can be modeled as an oscillator:

$$\ddot{x} + 4\dot{x} + 5x = F\cos(\omega_f t)$$

(a) What is the general solution to the associated homogeneous problem: $\ddot{x} + 4\dot{x} + 5x = 0$?

(b) Consider the case when your willpower provides a constant amount of "force", $F > 0$, to keep you feeling good. In this case, your mood is modeled by

$$\ddot{x} + 4\dot{x} + 5x = F\,, \qquad F > 0$$

(i) Find the general solution to this nonhomogeneous equation.
(ii) What is the long time behavior of the solution? That is, what is $\lim_{t\to\infty} y(t)$?

(c) Now suppose your willpower provides a varying amount of "force", $F(t) = \cos t$. Find the solution to the associated IVP given by

$$\ddot{x} + 4\dot{x} + 5x = \cos t\,, \qquad x(0) = 1\,, \qquad \dot{x}(0) = 0$$

Problem 5: (40 points)
Consider the differential equation

$$\ddot{x} + b\dot{x} + \frac{9}{4}x = 0 \tag{1}$$

(a) Find the roots of the characteristic polynomial for this differential equation.

(b) Provide a general solution for the differential equation. Note: There are two values of the parameter b for which the differential equation is different than all other values of b. Be sure to address this case separately.

(c) Describe all different possible behaviors of the solution to this differential equation based upon the value of b. That is to say, give values of b such that
(i) the solution is or is not oscillatory
(ii) the solution grows or decays