APPM 2360: Midterm Exam 2

June 29, 2018

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your recitation section number and (4) a grading table. Textbooks, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **1 side only**) note sheet is allowed.

Problem 1: (40 points) Answer each part of this question with TRUE or FALSE. **DO NOT** write T or F. Also, **provide a brief ONE SENTENCE justification.**

(a) The set of continuous functions on $[0, 1]$ satisfying $f(0) = 0$ is a subspace of the set of continuous functions on $[0, 1]$.

(b) For any $n \times n$ matrices A and B, $|AB| = |BA|$.

(c) Suppose A and B are both invertible $n \times n$ matrices. Then the solution to $BA\vec{x} = \vec{b}$ is $\vec{x} = B^{-1}A^{-1}\vec{b}$.

(d) If A is a 3×4 matrix with 3 pivots, then there exist $\vec{b} \in \mathbb{R}^3$ such that $A\vec{x} = \vec{b}$ has no solution.

(e) Three linearly independent vectors in $\mathbb{R}^3$ always span $\mathbb{R}^3$.

Problem 2: (40 points) The following problems are unrelated.

(a) Determine whether the following matrix is invertible or not. State explicitly whether the matrix is invertible or singular.

$$A = \begin{bmatrix} 0 & 3 & 5 \\ 5 & 1 & 2 \\ 3 & 2 & 3 \end{bmatrix}$$

(b) Determine whether $W = \{2 \times 2 \text{ matrices with determinant equal to } 0\}$ is a subspace of the vector space $V = \mathbb{M}_{2\times 2} = \{2 \times 2 \text{ matrices}\}$

Problem 3: (40 points)

(a) Given the matrix $A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 1 & 2 \\ 3 & 2 & 5 \end{bmatrix}$ and the vector $\vec{b} = \begin{bmatrix} 1 \\ -2 \\ -1 \end{bmatrix}$

find **ONE** solution to the equation $A\vec{x} = \vec{b}$.

(b) Let B be a 3×3 matrix of real numbers. The equation $B\vec{x} = \vec{c}$ is satisfied by $\vec{x}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\vec{x}_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$, for some vector $\vec{c}$. Suppose B has two pivot columns when in reduced row echelon form. Find **ALL** solutions to $B\vec{x} = \vec{0}$.

Problem 4: (40 points)

Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 1 & 1 & 0 & 2 \\ 3 & 2 & -1 & 5 \end{bmatrix}$$

(a) Find a basis for the column space of A, the span of the columns of A. That is to say, find a basis for the vector space of all vectors $\vec{b}$ such that the equation $A\vec{x} = \vec{b}$ is consistent.

(b) Find a basis for the null space of A. That is to say, find a basis for the vector space of all vectors $\vec{x}$ such that the equation $A\vec{x} = \vec{0}$ is satisfied.

(c) Briefly verify that any linear combination of the basis vectors given in part (b) satisfies $A\vec{x} = \vec{0}$.

Problem 5: (40 points) The following problems are unrelated.

(a) For what value(s) of a and b, if any, can you conclude that the following set of functions is linearly independent for $t \in \mathbb{R}$

$$\left\{e^{at},\ e^{bt},\ 1\right\}$$

(b) (i) Determine whether the following set of vectors is linearly independent or linearly dependent.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \right\}$$

(ii) Find a basis for and the dimension of

$$\text{span} \left\{ \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \right\}$$

Note that the vectors here are the same ones as in part (i).