- This exam is worth 100 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/021424 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The differential equation $x'(t) = -2(t-1)(x+1)^2$ has two equilibrium solutions.

   (b) Suppose L is a linear operator and $y(t), z(t)$ and $w(t)$ are functions such that $L[y(t)] = 3e^{-t}, L[z(t)] = 0$ and $L[w(t)] = -\sin t$. Then $L[y(t) - 3w(t) + 2z(t)] = 3\left(e^{-t} + \sin t\right)$.

   (c) $w(x) = xe^x$ is a solution to $xw' - w = x^2e^x$ on the entire real line.

   (d) Picard's theorem can be used to show that the initial value problem $y' = \sqrt[3]{ty},\ y(1) = 0$ has a nonunique solution.

   (e) The isoclines of the equation $y' - 2(t-y) + 1 = 0$ are a family of lines all having the same slope.

2. [2360/021424 18 pts)] Use the integrating factor method to solve the following initial value problem, identifying the transient and steady state solutions, if any exist.

$$\frac{t}{2}Q' + Q = \frac{\cos 2t}{t} + 3,\ Q(\pi) = 4,\ t > 0$$

3. [2360/021424 (19 pts)] The population of a certain species is given by $S(t)$, where t is measured in years and $S(t)$ is measured in hundreds of individuals, that is, $S = 2$ implies that there are 200 individuals present. The evolution of the population is governed by the equation $S' = 2tS - 4t$ and there are 100 individuals initially. Use the Euler-Lagrange two stage method (variation of parameters) to determine if the species will go extinct in a finite amount of time, t_f. Find t_f or explain why the species does not go extinct.

4. [2360/021424 (18 pts)] Euler-homogeneous equations, studied in the homework, are nonseparable equations of the form $y' = f(y/x)$. Recall that these can be made separable by a change of variable, $v = y/x$. Use this information to find the explicit general solution of $2xyy' - x^2 - 3y^2 = 0$.

5. [2360/021424 (17 pts) Consider the differential equation $w' = w^4 - 9w^2$.

   (a) (5 pts) Suppose one step of Euler's Method is applied to the initial value problem consisting of the differential equation and the initial condition $w(0) = w_1 = 1$, yielding the approximation $w_2 = \frac{1}{2}$. What stepsize was used to compute this approximation?

   (b) (10 pts) Find all of the equilibrium solutions and determine their stability. Plot the phase line.

   (c) (2 pts) For what initial values of w will solutions be bounded as $t \to \infty$?

6. [2360/021424 (18 pts)] Consider the following Lotka-Volterra predator-prey equations, where x represents the prey and y the predator.

$$\frac{\mathrm{d}x}{\mathrm{d}t} = 100x - 20xy$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -60y + 20xy$$

   (a) (4 pts) Find the h nullcline(s).

   (b) (4 pts) Find the v nullcline(s).

   (c) (4 pts) Find all the equilibrium points, if any exist.

   (d) (6 pts) Determine if the predator and prey populations are increasing, decreasing or remaining constant at the following points:

   i. $(5,5)$ ii. $(3,10)$ iii. $(1,2)$