1. Given the matrices
   \[ A = \begin{bmatrix} 1 & 0 \\ 3 & 4 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -1 & -3 \\ 0 & 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 4 \end{bmatrix} \]
   write the word TRUE or FALSE as appropriate. No work need be shown, no work will be graded and no partial credit will be given.
   
   (a) \( CB = \begin{bmatrix} -2 \\ 5 \\ 11 \end{bmatrix} \)
   (b) \( \text{Tr} (B^T A^T) = 2 \)
   (c) \( A^T A = AA^T \)
   (d) \( |C^T C - 3I| = -10 \)
   (e) \( AB - A^T B^T \) is not defined

2. Let \( A = \begin{bmatrix} 0 & 0 & 3 \\ 0 & 3 & 0 \\ 3 & 0 & 1 \end{bmatrix} \).
   
   (a) (4 pts) Find the eigenvalues of \( A \) and state the multiplicity (also known as the algebraic multiplicity) of each.
   
   (b) (8 pts) Find the dimension of and a basis for the eigenspace associated with the eigenvalue whose (algebraic) multiplicity is greater than 1.

3. Let \( \vec{p}_1 = 1 + x^2, \vec{p}_2 = x - x^2, \vec{p}_3 = 2 + 2x + 4x^2 \). Show that \( \vec{p} = 3 + 4x - 2x^2 \) is in span \( \{ \vec{p}_1, \vec{p}_2, \vec{p}_3 \} \) by writing \( \vec{p} \) as a linear combination of \( \vec{p}_1, \vec{p}_2, \vec{p}_3 \). Use Cramer’s Rule and cofactor expansion to solve an appropriate linear system.

4. Let \( A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & 1 \end{bmatrix} \). NO credit will be given if Gauss-Jordan elimination is used.
   
   (a) (5 pts) Using only matrix multiplication, verify that \( B = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 3 & 0 & -2 \end{bmatrix} \) is the inverse of \( A \).
   
   (b) (9 pts) Using only matrix multiplication and properties of the matrix inverse and transpose, solve \( A^T A \vec{x} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \).

5. Determine if each of the following sets of vectors forms a basis for \( \mathbb{R}^3 \). Justify your answers.
   
   (a) \( \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} \right\} \)
   (b) \( \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 8 \\ -2 \end{bmatrix} \right\} \)
6. [2360/030922 (24 pts)] The following parts are unrelated.

(a) (12 pts) Find the RREF of \( A = \begin{bmatrix} 1 & 3 & 1 & 9 \\ 1 & 1 & -1 & 1 \\ 3 & 11 & 5 & 35 \end{bmatrix} \).

(b) (12 pts) We need to solve the system \( A \vec{x} = \vec{b} \). After a number of elementary row operations, the augmented matrix for the system is

\[
\begin{bmatrix}
1 & 0 & 0 & 3 & 5 \\
0 & 1 & 3 & 0 & -2 \\
0 & 0 & 0 & 1 & -2 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

i. (10 pts) Use this and the Nonhomogeneous Principle to find the solution to the original system.

ii. (2 pts) Find the dimension of the solution space of the original associated homogeneous system, \( A \vec{x} = \vec{0} \). Hint: You have the information you need from part (i); very little additional work is required.

7. [2360/030922 (14 pts)] Determine if the subsets, \( \mathcal{W} \), are subspaces of the given vector spaces, \( \mathcal{V} \).

(a) (7 pts) \( \mathcal{V} = \mathbb{M}_{22} \); \( \mathcal{W} = \{ A \in \mathbb{M}_{22} \mid A^T = -A \} \), the set of all matrices of the form \( \begin{bmatrix} 0 & k \\ -k & 0 \end{bmatrix} \) where \( k \) is a real number.

(b) (7 pts) \( \mathcal{V} = \mathbb{R}^3 \); \( \mathcal{W} = \left\{ \vec{v} \in \mathbb{R}^3 \mid \begin{bmatrix} p + q \\ r \\ s \end{bmatrix} \text{ where } p, q, r, s \in \mathbb{R} \text{ and } s \geq 0 \right\} \).