- This exam is worth 100 points and has 6 questions.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on one sides.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/020922 (28 pts)] Consider the initial value problem $y' + y = e^t$, $y(0) = 1$.

   (a) (2 pts) Write all the numbers that apply when classifying the differential equation:
   i. separable ii. linear iii. autonomous iv. homogeneous

   (b) (8 pts) Use Euler's Method to approximate $y(\ln 4) = y(2\ln 2)$ using a stepsize of $h = \ln 2$.

   (c) (6 pts) Draw the isoclines where the slope of the solution curve is $1, -1, 0$. Include line segments on each isocline indicating the slope. Label any intercepts.

   (d) (12 pts) Use the Euler-Lagrange Two Stage method (variation of parameters) to solve the initial value problem.

2. [2360/020922 (19 pts)] Consider the differential equation $y' + y^2 - 2y + 1 = 0$.

   (a) (2 pts) Write all the numbers that apply when classifying the differential equation:
   i. separable ii. linear iii. autonomous iv. homogeneous

   (b) (3 pts) Find all equilibrium solutions of the differential equation.

   (c) (3 pts) Draw the phase line for the differential equation.

   (d) (2 pts) Determine the stability of all equilibrium solutions.

   (e) (4 pts) Show that $y = 1 + (c+t)^{-1}$ is a solution to the differential equation.

   (f) (5 pts) The differential equation can be interpreted as a model of logistic growth with harvesting, where $y > 0$ is the number of fish (in thousands) and t is measured in years. If the initial $(t = 0)$ number of fish is 500, will the fish go extinct? If so, when? If not, explain why not.

3. [2360/020922 (8 pts)] Consider the initial value problem (IVP) $y' + t(y+1)^{1/3} = y^2 \cos t$, $y(-1) = -1$. Does Picard's Theorem guarantee that the IVP has a unique solution? Justify your answer.

4. [2360/020922 (17 pts)] A 100 gallon tank is initially 70 percent full of vinegar in which 10 grams of the ferocious Spice 2360 is dissolved to make a tangy barbecue sauce. In both parts (a) and (b) set up, but **DO NOT SOLVE**, the initial value problem (IVP) that models the given scenario. Be sure to define your variables.

   (a) (7 pts) Vinegar containing 6 grams per gallon of the spice flows into the tank at 2 gallons per hour and the well mixed sauce leaves the tank at 2 gallons per hour.

   (b) (7 pts) Pure vinegar enters the tank at 4 gallons per hour and the well mixed sauce leaves the tank at 2 gallons per hour.

   (c) (3 pts) Over what time interval will the solution to part (b) be valid? You do not need to find the solution.

5. [2360/020922 (16 pts)] Let $t^2 x' + t(t+2)x - e^{7t} = 0$.

   (a) (2 pts) Write all the numbers that apply when classifying the differential equation:
   i. separable ii. linear iii. autonomous iv. homogeneous

   (b) (12 pts) Use the integrating factor method to find the general solution of the differential equation.

   (c) (2 pts) Determine the largest interval over which the general solution is defined.

6. [2360/020922 (12 pts)] Consider the system of differential equations

$$x' = 4 - x^2 - y$$
$$y' = -8 + 2x^2 - y$$

(a) On the same graph,

i. (3 pts) Plot the v nullclines, if any, as solid curve(s). Label all intercepts

ii. (3 pts) Plot the h nullclines, if any, as dashed curve(s). Label all intercepts.

iii. (2 pts) Draw an arrow (an element of the vector field) indicating the direction of the trajectory at the point $(0, 4)$.

iv. (2 pts) Draw an arrow (an element of the vector field) indicating the direction of the trajectory at the point $(0, -8)$.

(b) (2 pts) Find all equilibrium solutions, if any.