1. Consider the system of differential equations

\[
x' = y + 3 \\
y' = 1 - x^2 - y
\]

On a single graph, plot the following items:
- vectors indicating the direction of the trajectories at the following points: \((\frac{3}{2}, -\frac{5}{4}), (\frac{1}{2}, -3)\)
- nullclines, if any, as dashed curves; be sure to label any intercepts that exist
- nullclines, if any, as solid curves; be sure to label any intercepts that exist
- all equilibrium points, if any exist

2. Let \(A = \begin{bmatrix} k & 0 & 0 & 0 \\ -1 & 3 & 0 & 0 \\ 5 & 4 & k-4 & 0 \\ 4 & -6 & 3 & -2 \end{bmatrix}\).

(a) For what value(s) of \(k\) will the linear system \(A \vec{x} = \vec{b}\) have a unique solution? \((\vec{b} is an arbitrary vector in \(\mathbb{R}^4\)). Justify your answer.

(b) For what value(s) of \(k\) will the linear system \(A \vec{x} = \vec{0}\) be consistent? Justify your answer.

(c) If \(k = -1\), find \(|A^{-1}|\) and \(|A^T|\).

(d) With \(k = 2\), use Cramer’s Rule to solve the linear system \(A \vec{x} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}\). Hint: take advantage structure of the matrices whose determinants you need to compute.

(e) Again letting \(k = 2\) and \(B = \frac{1}{24} \begin{bmatrix} 12 & 0 & 0 & 0 \\ 4 & 8 & 0 & 0 \\ 38 & 16 & -12 & 0 \\ 69 & 0 & -18 & -12 \end{bmatrix}\), find \(BA\) and describe the relationship between \(A\) and \(B\).

3. The following problems are not related.

(a) Let \(W\) be the set of \(3 \times 3\) matrices with trace equal to 1. Is \(W\) a subspace of \(M_{33}\)? Justify your answer.

(b) Is the set \(E\) of even functions \(\{f \mid f(-x) = f(x)\}\) defined on the entire real line a subspace of \(C(-\infty, \infty)\)? Justify your answer, assuming the standard operations of addition and scalar multiplication.

4. The following problems are not related.

(a) Find a basis for and the dimension of the solution space of the linear system \(A \vec{x} = \vec{0}\) whose augmented matrix is \(\begin{bmatrix} 1 & 0 & -1 & 4 & 0 \\ 0 & 1 & -2 & 1 & 0 \end{bmatrix}\).

(b) Find the RREF of the matrix \(\begin{bmatrix} -2 & -4 & -8 \\ 1 & 2 & 4 \\ 7 & 2 & 4 \end{bmatrix}\) and determine the number of pivot columns.

(c) Consider the set of functions \(\{-7, 5t^2 + t, 1 - 2t - 10t^2\}\) in the vector space \(P_2\).
   i. Show that the Wronskian of the functions in the set is 0.
   ii. Are the functions linearly dependent or linearly independent on the real line? Justify your answer. [Hint: part (i) does not help you answer this question]
   iii. Do the functions form a basis for \(P_2\)? Briefly explain.

5. Given \(A = \begin{bmatrix} 1 & 0 & -1 \\ -3 & -2 & 3 \end{bmatrix}\) and \(B = \begin{bmatrix} 2 & -3 \\ 3 & -4 \end{bmatrix}\), compute the following, if possible, or write NOT DEFINED.

(a) \(AB\)  (b) \(A^T B\)  (c) \(|A|\)  (d) \(\text{Tr}(BB^T)\)  (e) \(A - 2A\)  (f) \((A^{-1})^T\)  (g) \((B + I)^2\)