APPM 2360: Midterm exam 3
April 17, 2019

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your recitation section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **2 sided**) crib sheet is allowed.

Problem 1: (30 points)

(a) (8 points) The numbers $r = 1, -2+i$ are characteristic roots of the 3rd order homogeneous differential equation of the form $y''' + by'' + cy' + dy = 0$.
Find the constant coefficients b, c and d.

(b) (14 points) Solve the IVP

$$y'' - 4y' + 8y = 0, \qquad y(0) = 0,\, y'(0) = 1.$$

(c) (8 points) Find the general solution to the differential equation $y^{(4)} - 2y''' + y'' = 0$.

Problem 2: (30 points) Solve the following problems:

(a) (10 pts). Find the general solution to

$$y'' - 2y' + y = 0.$$

(b) (10 pts). Give the general form of the solution to the nonhomogeneous equation according to the method of undetermined coefficients (**do not solve for coefficients**):

$$y'' - 2y' + y = 6 + te^t.$$

(You may use your answer from part (a) of this problem.)

(c) (10 pts). For what values of ω will the following harmonic oscillator system exhibit resonance?

$$x''(t) + \omega^2 x(t) = 3\cos(t).$$

Problem 3: (30 points) Consider the following differential equation: $y'' - y = e^t$. The solution to $y'' - y = 0$ is $y_h = c_1 y_1 + c_2 y_2 = c_1 e^t + c_2 e^{-t}$.

(a) When looking for a particular solution y_p using variation of parameters, what is the general form of y_p?

(b) Solve for the varying parameters $v_1(t)$ and $v_2(t)$.

(c) Write the specific form of y_p.

(d) Write the general solution y and simplify it as much as possible.

(e) Does the method of undetermined coefficients apply to this differential equation? If so, what is the appropriate form of y_p? Explain. Does this agree with your answer above?

Problem 4: (30 points)

(a) (5 points) Use the **integral definition** (do not use a table) to calculate the Laplace transform $F(s)$ of the function $f(t) = e^{-3t}$. For which s is it defined?

(b) (5 points) Use a table and properties of the Laplace transform to find $\mathcal{L}^{-1}\left\{\dfrac{-2s+6}{s^2+4}\right\}$.

(c) (20 points) Use the Laplace transform to solve the IVP

$$y'' - 3y' + 2y = e^{3t}, \quad y(0) = 1, \quad y'(0) = 5$$

Problem 5: (30 points) **True/False** (answer True if it is always true, otherwise answer False). No justification is required as there is no partial credit on this question.

(a) (6 points) For some parameters, the damped oscillator equation $my'' + by' + ky = 0$ has some solutions like $y(t) = t\mathrm{e}^{-at}$, but for other parameters, it can have solutions like $y(t) = \mathrm{e}^{-at}\cos(\omega t)$.

(b) (6 points) For all choices of coefficients a, b, c, particular solutions of the differential equation $ay'' + by' + cy = t^2$ are of the form $y_p(t) = At^2 + Bt + C$.

(c) (6 points) The amplitude of solutions to $y'' + 2y' + 2y = \cos(t)$ grows indefinitely.

(d) (6 points) The Laplace transform $\mathcal{L}$ of $f(t) = 0$ cannot be defined.

(e) (6 points) The initial value problem $y'' + y = \mathrm{e}^t$ with $y(0) = 1$ and $y'(0) = 0$ can be solved using method of undetermined coefficients, variation of parameters, or Laplace transforms, and the resulting solutions are all the same.

Short table of Laplace Transforms

$$\mathcal{L}\{1\} = \frac{1}{s} \qquad \mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}} \qquad \mathcal{L}\{e^{at}\} = \frac{1}{s-a} \qquad \mathcal{L}\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$$

$$\mathcal{L}\{\cos(bt)\} = \frac{s}{s^2+b^2} \quad \mathcal{L}\{\sin(bt)\} = \frac{b}{s^2+b^2} \quad \mathcal{L}\{\cosh(bt)\} = \frac{s}{s^2-b^2} \quad \mathcal{L}\{\sinh(bt)\} = \frac{b}{s^2-b^2}$$

$$\mathcal{L}\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2+b^2} \qquad \mathcal{L}\{e^{at}\sin(bt)\} = \frac{b}{(s-a)^2+b^2}$$

$$\mathcal{L}\{f^{(n)}(t)\} = s^n\mathcal{L}\{f(t)\} - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$$