APPM 2360: Midterm exam 2
March 13, 2019

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your recitation section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **2 sided**) crib sheet is allowed.

Problem 1: (30 points). Consider the following matrix:

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 1 & \beta \\ 0 & 0 & \alpha \end{bmatrix}$$

(a) (6 points) Find the eigenvalues of the matrix A.
(b) (6 points) For which values of α and β does A^{-1} exist?
(c) (12 points) Find the eigenvectors of the matrix A provided all the eigenvalues are distinct.
(d) (6 points) How many linearly independent eigenvectors can be associated with $\alpha = 1$?

Problem 2:(30 points) **True/False** (answer True if it is always true, otherwise answer False). No justification is required as there is no partial credit on this question.

(a) (6 points) The set $S = \{x^3, x^2 + x + 1, x - 1\}$ is a basis for $\mathbb{P}_3$, the vector space of all polynomials of degree less than or equal to 3.
(b) (6 points) The following set of vectors form a basis for $\mathbb{R}^4$: $\begin{pmatrix} 10 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 5 \\ 6 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 7 \\ 8 \\ 9 \\ 0 \end{pmatrix}$
(c) (6 points) The matrix $\begin{bmatrix} 1 & 2 \\ 2 & k \end{bmatrix}$ is invertible for all $k \neq 0$
(d) (6 points) If the reduced-row echelon form (RREF) of a matrix A is $R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $|A| = 1$.
(e) (6points) If $|A| = 5$, then $|B^{-1}AB| = 5$

Problem 3: (30 points) Provide clear justification for each of the unrelated problems listed below. **No points will be awarded if there is no justification.**

(a) (9 pts) Are the functions $f_1(t) = \sin(t)$ and $f_2(t) = \sin(2t)$ linearly independent for $t > 0$? Explain why or why not?
(b) (7 pts) Let $\mathbb{V}$ be the vector space of continuous functions that have derivatives of all orders on $[0, 1]$ and $\mathbb{W} = \{f(t), t \in [0, 1] | f'(t) \geq 0\}$ (the set of functions with a first derivative equal to or greater than zero). Is $\mathbb{W}$ a vector subspace of $\mathbb{V}$?
(c) (7 pts) Let $\mathbb{P}_4$ be the vector space of polynomials in x of order 4 or less and $\mathbb{W} = \{kx^4 + x^2 | k \in \mathbb{R}\}$. Is $\mathbb{W}$ a vector subspace of $\mathbb{P}_4$?
(d) (7 pts) Let $\mathbb{V}$ be the vector space of continuous functions that have three continuous derivatives on $\mathbb{R}$ and $\mathbb{W} = \{y(t), t \in \mathbb{R} : y''' + \sin(t) y' = 3y\}$. Is $\mathbb{W}$ a vector subspace of $\mathbb{V}$?

Problem 4: (30 points) The following problems are not related.

(a) (10 pts) Write the following system of algebraic equations in matrix-vector form.

$$\begin{cases} 2x + 3y + 2z = 5, \\ 2z - y = 0, \\ 2x - 4z = 1. \end{cases}$$

(b) (20 pts) Find all solutions to the following system of algebraic equations using row-operations. Separate the solution out into a sum of the homogeneous and particular solutions.

$$\begin{pmatrix} 3 & 1 & -1 \\ 1 & 0 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}.$$

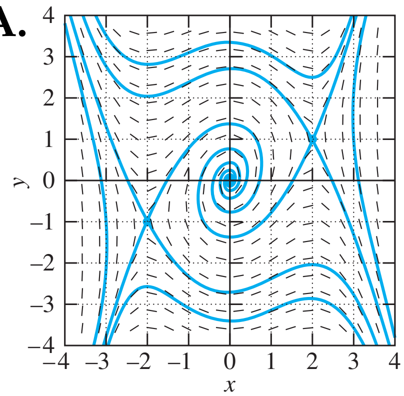
Problem 5: (30 points) Solve the following unrelated problems involving systems of differential equations.

(a) (20 pts) Match each system to its phase portrait. You do not need to show any work:

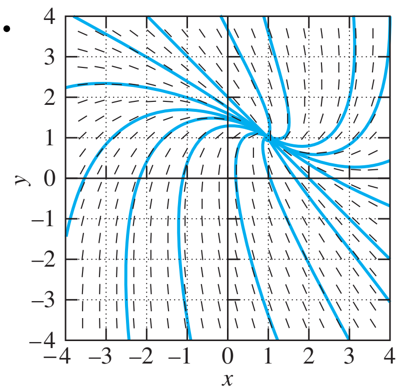
$(i)\ \dot{x} = x - y, \quad \dot{y} = x + 3y - 4, \qquad (ii)\ \dot{x} = x - 2y + 3, \ \ \dot{y} = x - y + 2,$

$(iii)\ \dot{x} = 2 - 4x - 15y, \ \ \dot{y} = 4 - x^2 \qquad (iv)\ \dot{x} = x - 2y, \ \ \dot{y} = 4x - x^3.$

A.



B.



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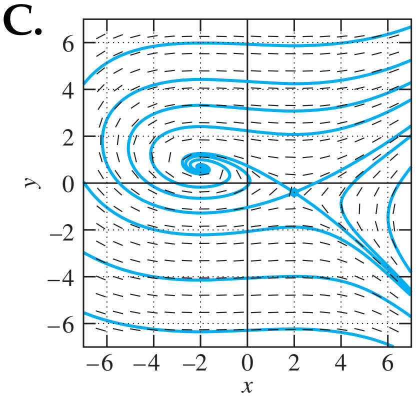

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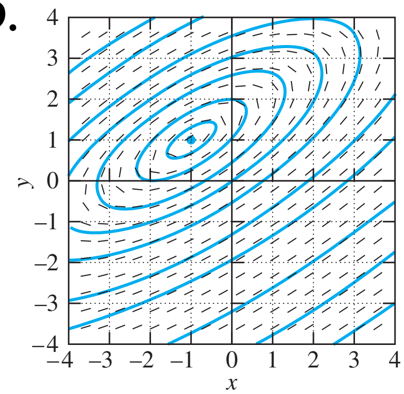

(b) (10 pts) Solve for the nullclines and fixed point(s) of the following system.

$$\dot{x} = -x + y, \qquad \dot{y} = 2x - y.$$

Plot and label the nullclines, fixed point(s), and two solution trajectories, and identify the stability of any fixed point(s).