- This exam is worth 100 points and has 7 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/102324 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) If $\mathbf{A}$ is 2×3 and $\mathbf{B}$ is 3×4, then $(\mathbf{AB})^{\mathrm{T}}$ is 2×4.

   (b) Given a vector space $\mathbb{V}$, any set of vectors in $\mathbb{V}$ containing $\vec{\mathbf{0}}$ is linearly dependent.

   (c) If $\mathbf{G}$ is an $n \times n$ matrix and m is a positive integer, then $|\mathbf{G}^m| = |\mathbf{G}|^m$.

   (d) Cramer's Rule can be used to solve $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ if $\mathbf{A}$ is 5×5 and rank $\mathbf{A} = 4$

   (e) If $\mathbf{A}$ is a 10×10 matrix with $|\mathbf{A}| = 2$ and $\mathbf{B}$ is the matrix obtained by multiplying row 4 of $\mathbf{A}$ by 4, then $|\mathbf{B}| = 8$.

2. [2360/102324 (14 pts)] Use Gauss-Jordan row reduction to find the inverse of an appropriate matrix and use that inverse matrix to find the solution of

$$\begin{aligned} x_1 + x_2 \qquad &= 1 \\ x_1 + x_2 + x_3 &= -2 \\ x_2 + x_3 &= -3 \end{aligned}$$

3. [2360/102324 (19 pts)] The following problems are not related.

   (a) (7 pts) What does the Wronskian tell you about the linear independence of the functions $\{x, 1+x, 2+3x\}$ on the real line?

   (b) (12 pts) Determine which of the following sets of vectors form a basis for the given vector space, $\mathbb{V}$. Justify your answers.

      i. (4 pts) $\mathbb{V} = \mathbb{R}^2$; $\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \end{bmatrix} \right\}$

      ii. (4 pts) $\mathbb{V} = \mathbb{R}^3$; $\left\{ \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 1 \end{bmatrix} \right\}$

      iii. (4 pts) $\mathbb{V} = \mathbb{P}_3$; any set of 3 linearly independent third degree polynomials

4. [2360/102324 (13 pts)] Consider the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 8 & 10 \\ -3 & -7 & -5 \end{bmatrix}$.

   (a) (10 pts) For what value(s) of k is $\vec{\mathbf{b}} = \begin{bmatrix} -1 & k & 1 \end{bmatrix}^{\mathrm{T}} \in$ Col $\mathbf{A}$?

   (b) (3 pts) For what value(s) of k is the system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$ inconsistent?

MORE PROBLEMS BELOW/ON REVERSE

5. [2360/102324 (20 pts)] Consider the following augmented matrix of the linear system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$:

$$\left[\begin{array}{rrr|r} 1 & 3 & 1 & -1 \\ -2 & -5 & -3 & -1 \\ 5 & 16 & 4 & -8 \\ 0 & -1 & 1 & 3 \end{array}\right]$$

(a) (2 pts) Is the system overdetermined, underdetermined or neither?

(b) (10 pts) Find the general solution, that is, solve the system.

(c) (8 pts) Find a basis for the solution space of the associated homogeneous system and find the dimension of that solution space. Your work from part (b) may prove beneficial.

6. [2360/102324 (12 pts)] Let $\mathbf{B} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

(a) (6 pts) Find the eigenvalues of $\mathbf{B}$ and state their algebraic multiplicity.

(b) (6 pts) Find a basis for the eigenspace associated with the largest eigenvalue you found in part (a). What is the geometric multiplicity of this largest eigenvalue?

7. [2360/102324 (12 pts)] Determine if the following subsets, $\mathbb{W}$, are subspaces of the given vector space, $\mathbb{V}$. Justify your answer.

(a) (6 pts) $\mathbb{V} = \mathbb{R}^2$, $\mathbb{W}$ is the set of vectors of the form $\begin{bmatrix} a \\ a^2 \end{bmatrix}$ where a is a real number.

(b) (6 pts) $\mathbb{V} = \mathbb{M}_{22}$, $\mathbb{W}$ is the set of 2×2 matrices for which the entries in each column sum to zero.