- This exam is worth 100 points and has 6 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. $\boxed{\textbf{NO}}$ calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.

---

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/102523 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   For all parts of the problem, assume $\mathbf{A}$ is a 2×5 matrix and $\mathbf{C}$ is a singular 5×5 matrix.

   (a) $\left|\left(\mathbf{A}\mathbf{A}^{\mathrm{T}}\right)^2\right| \neq \left|\mathbf{A}\mathbf{A}^{\mathrm{T}}\right|^2$

   (b) 0 is an eigenvalue of $\mathbf{C}$.

   (c) $\mathbf{C}\vec{\mathbf{x}} = \vec{\mathbf{0}}$ is inconsistent.

   (d) $\mathbf{A} + \left(\mathbf{C}\mathbf{A}^{\mathrm{T}}\right)^{\mathrm{T}}$ is not defined.

   (e) $(\mathrm{Tr}\ \mathbf{C})^2 - 4|\mathbf{C}| = (\mathrm{Tr}\ \mathbf{C})^2$

2. [2360/102523 (25 pts)] Let $\mathbf{B} = \begin{bmatrix} 2 & 0 & 0 \\ 4 & -1 & 0 \\ 6 & 3 & 1 \end{bmatrix}$.

   (a) (10 pts) Use Gauss-Jordan elimination to find $\mathbf{B}^{-1}$.

   (b) (5 pts) State the definition for $n \times n$ matrix $\mathbf{H}$ to be the inverse of $n \times n$ matrix $\mathbf{G}$ (there are two requirements). Verify that your answer to part (a) is correct by showing that one of the requirements in the definition holds.

   (c) (6 pts) Use your answer to part (a) to solve the system $\mathbf{B}\vec{\mathbf{y}} = \vec{\mathbf{c}}$ where $\vec{\mathbf{c}} = \begin{bmatrix} 4 & 10 & 6 \end{bmatrix}^{\mathrm{T}}$.

   (d) (4 pts) Is Col $\mathbf{B} = \mathbb{R}^3$? Explain briefly in words.

3. [2360/102523 (16 pts)] Consider the system

$$x_1 + 6x_2 - 2x_3 + 3x_4 = 5$$

$$3x_1 + 17x_2 - 6x_3 + 9x_4 = 13$$

   (a) (8 pts) Find the solution to the system, writing your answer using the Nonhomogeneous Principle.

   (b) (8 pts) Find a basis for the solution space of the associated homogeneous system. What is its dimension?

4. [2360/102523 (15 pts)] Let $\mathbf{F} = \begin{bmatrix} 1 & 3 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$

   (a) (5 pts) Using the definition of what it means to be an eigenvalue/eigenvector pair (do not use a determinant), find the eigenvalue associated the eigenvector $\vec{\mathbf{v}}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

   (b) (10 pts) The other eigenvalue is -2. What is its algebraic multiplicity? Find its geometric multiplicity as well as a basis for and the dimension of its eigenspace.

CONTINUED ON REVERSE

5. [2360/102523 (18 pts) Determine which of the following subsets of $\mathbb{M}_{33}$ are subspaces. Justify your answers.

   (a) (6 pts) The subset, $\mathbb{W}$, of matrices of the form $\begin{bmatrix} a & 0 & a \\ 0 & a+2 & 0 \\ a & 0 & a \end{bmatrix}$ where $a \in \mathbb{R}$.

   (b) (6 pts) The subset, $\mathbb{W}$, of 3×3 skew symmetric matrices $\left(\mathbf{A}^{\mathrm{T}} = -\mathbf{A}\right)$.

   (c) (6 pts) The subset, $\mathbb{W}$, of 3×3 upper triangular matrices with rational number entries.

6. [2360/102523 (16 pts)] The following parts are not related. Both parts require complete justification.

   (a) (8 pts) Is the set $\left\{e^t, t, t^2\right\}$ linearly dependent or independent on the real line?

   (b) (8 pts) Does span $\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix} \right\} = \mathbb{R}^3$?