- This exam is worth 100 points and has 5 questions.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/101922 (24 pts)] The following parts are not related.

   (a) (10 pts) Show that the Wronskian of the functions $\left\{t+3, t^2-1, 2t^2-t-5\right\}$ cannot be used to determine if the functions are linearly dependent or independent.

   (b) (14 pts) Consider the linear system

$$x_1 + 2x_2 + x_3 = 4$$
$$x_1 + 2x_2 = 2$$
$$x_1 + x_2 + 3x_3 = -7$$

   Use the inverse of the coefficient matrix to find the solution to the system. You must use elementary row operations to find the inverse.

2. [2360/101922 (20 pts)] The following parts are not related. Justify your answers.

   (a) (10 pts) Does the set $\left\{\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}2\\1\\-1\end{bmatrix}\right\}$ form a basis for $\mathbb{R}^3$?

   (b) (10 pts) Determine if the following subsets $\mathbb{W}$ of the given vector space $\mathbb{V}$ are subspaces. Assume that the standard definitions of vector addition and scalar multiplication apply.

      i. (5 pts) $\mathbb{V} = \mathbb{M}_{33}$; $\mathbb{W}$ is the set of matrices of the form $\begin{bmatrix} r & 0 & p \\ 0 & s & 0 \\ q & 0 & t \end{bmatrix}$ where p, q are real numbers and r, s, t are integers.

      ii. (5 pts) $\mathbb{V} = \mathbb{R}^3$; $\mathbb{W}$ is the set of solutions to the equation $4x_1 - 3x_2 + 9x_3 = 0$.

3. [2360/101922 (12 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) For any matrix $\mathbf{A}$, $\left|\mathbf{A}^{\mathrm{T}}\mathbf{A}\right|$ and $\left|\mathbf{A}\mathbf{A}^{\mathrm{T}}\right|$ are defined.

   (b) For invertible matrices $\mathbf{A}$ and $\mathbf{B}$, $\left|\mathbf{A}\mathbf{B}^{-1}\right| = |\mathbf{A}|/|\mathbf{B}|$.

   (c) If square matrix $\mathbf{A}$ has 0 for an eigenvalue, then Cramer's Rule can be used to solve the system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{b}}$.

   (d) Tr $\left[(2\mathbf{I})^3\right] = 24$ where $\mathbf{I}$ is the 3×3 identity matrix.

   (e) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \in \text{span}\left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\right\}$

   (f) Homogeneous systems of linear algebraic equations are never inconsistent.

4. [2360/101922 (22 pts)] Consider the augmented matrix $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{array}\right]$ derived from the linear system $\mathbf{A}\vec{\mathbf{x}} = \vec{\mathbf{0}}$.

   (a) (10 pts) If the matrix is in RREF, write the word YES. Otherwise, write NO and put the matrix in RREF.

   (b) (12 pts) Find a basis for the solution space of the system. What is the dimension of the solution space?

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5. [2360/101922 (22 pts)] The following parts are not related.

(a) (10 pts) The characteristic polynomial of a certain matrix $\mathbf{B}$ is $p(\lambda) = 2\lambda^6 + 6\lambda^5 + 5\lambda^4$.

i. (2 pts) What is the order/size of the matrix $\mathbf{B}$?

ii. (8 pts) Find the eigenvalues of $\mathbf{B}$ and their (algebraic) multiplicities. Do not find the eigenvectors.

(b) (12 pts) Let $\mathbf{A} = \begin{bmatrix} -7 & 4 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 4 & -7 & -1 \\ 0 & 0 & 0 & -7 \end{bmatrix}$ and let $\mathbb{E}_{-7}$ denote the eigenspace associated with eigenvalue $\lambda = -7$. Find a basis for and the dimension of $\mathbb{E}_{-7}$.