- This exam is worth 100 points and has 5 questions.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted. You are allowed one 8.5"× 11" crib sheet with writing on one side.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2360/092122 (35 pts)] Consider the initial value problem $(t+1)y' - 3(t+1)y + e^{3t} = 0$, $y(0) = \ln 3$, $t > -1$.

   (a) (4 pts) Classify the equation.

   (b) (2 pts) Does the equation possess any equilibrium solutions? If so, find them.

   (c) (7 pts) Is a unique solution guaranteed by Picard's Theorem? Justify your answer.

   (d) (7 pts) Use one step of Euler's Method to approximate y when $t = 1/10$. Simplify your answer.

   (e) (15 pts) Suppose the equation describes the amount of water in a well (in millions of gallons) with t the time in years. Will the well run dry? If so, when. If not, explain why not.

2. [2360/092122 (15 pts)] Consider the differential equation $y' - y^2 + y^3 = 0$.

   (a) (2 pts) Classify the equation.

   (b) (4 pts) Find the equilibrium solutions.

   (c) (5 pts) Plot the phase line.

   (d) (2 pts) Determine the stability of all equilibrium solutions.

   (e) (2 pts) Find the solution that passes through $(1, 1)$. Hint: Very little work is required to answer this.

3. [2360/092122 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) Solutions to the differential equation $y' = -x^2 - y^4 - 3$ are always decreasing.

   (b) The operator $L(\vec{\mathbf{y}}) = 2y + (1-y)y^{(5)}$ is a linear operator

   (c) The isocline of the differential equation $x' - e^t + 1 = 0$ corresponding to a slope of -2 does not exist.

   (d) The substitution $u = y - 2x + 3$ makes the differential equation $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2 + \sqrt{y - 2x + 3}$ separable.

   (e) The system of differential equations below has a single equilibrium solution at $(0, 0)$.

$$x' = 2 - x^2 - y^2$$

$$y' = y^2 - x$$

4. [2360/092122 (15 pts)] A 1000 gallon pot is initially 80 percent full of sweet tea in which 100 ounces of sugar is dissolved. Tea containing $1/(t+1)$ ounces of sugar per gallon enters the pot at 5 gallons per minute. The well-mixed sweet tea leaves the pot at 7 gallons per minute.

   (a) (12 pts) Set up, but **DO NOT SOLVE**, an initial value problem for the amount of sugar, S, contained in the pot at time t.

   (b) (3 pts) If the initial time is $t = 0$, over what interval will the solution be valid? You do not need to find the solution to answer this question.

5. [2360/092122 (25 pts)] Consider the initial value problem $x\dfrac{\mathrm{d}w}{\mathrm{d}x} + (2x+1)w = 2x^2$, $w(1) = \dfrac{3}{2}$, $x > 0$.

   (a) (5 pts) Without solving the differential equation, show that $w_p(x) = \dfrac{1}{2x} + x - 1$ is a particular solution.

   (b) (15 pts) Find the general solution to the differential equation.

   (c) (5 pts) Solve the initial value problem.