1. [APPM 2360 Exam (18 pts)] Let \( L(y) = t^2 y'' - ty' + y \) with \( t > 1 \).
   (a) (6 pts) Show that the set \( \{t, t \ln t\} \) forms a basis for the solution space of \( L(y) = 0 \).
   (b) (12 pts) Find the general solution of \( L(y) = \frac{t}{\ln t} \). (Hint: \( u \)-substitution)

2. [APPM 2360 Exam (37 pts)] The following problems (a) and (b) are not related.
   (a) (22 pts) Consider the linear operator \( L(y) = ay^{(4)} + by''' + cy'' + py' + qy \) where \( a, b, c, p, q \) are real constants.
      i. (4 pts) If the characteristic equation of \( L(y) = 0 \) has the root 1 of algebraic multiplicity 2 and the roots \( 2 \pm 3i \), write the general solution of \( L(y) = 0 \).
      ii. (18 pts) For \( f(t) \) in each of the following, write the form (DO NOT SOLVE FOR THE CONSTANTS) of the particular solution that will be used for the method of undetermined coefficients when solving \( L(y) = f(t) \). If the method of undetermined coefficients is not applicable, write N/A.
         A. \( f(t) = 2t^{-1} \)  
         B. \( f(t) = e^t + \frac{t}{\pi} \)  
         C. \( f(t) = te^t + \sin t \)  
         D. \( f(t) = \sin 4t + e^{2t} \cos 3t \)  
         E. \( f(t) = t^3 e^{t/2} \)  
         F. \( f(t) = \tan 3t + 2 \)
   (b) (15 pts) Use the method of undetermined coefficients to solve the initial value problem
      \[ y''' - 2y'' = 4, \quad y(0) = 9, \quad y'(0) = 1, \quad y''(0) = 14 \]

3. [APPM 2360 Exam (24 pts)] The following problems (a) and (b) are not related.
   (a) (3 pts) Find the value(s) of \( b \) and \( m \) for which the harmonic oscillator governed by \( m\ddot{x} + b\dot{x} + x = \cos(2\pi t) \) will have unbounded solutions.
   (b) (21 pts) A 3 kg mass is attached to a spring which in turn is attached to a wall. The entire apparatus is on a horizontal shelf and immersed in a fluid that offers a damping force equal to 6 times the instantaneous velocity. The oscillator is unforced and has a Spring-o-Matic dial marked with values of \( k \) that allows you to adjust the spring (restoring) constant, \( k \).
      i. (6 pts) As the Spring-o-Matic dial is moved from \( k = 5 \) to 3 to 1, identify the type of damping (critical, over, under) that occurs for each of these values of \( k \).
      ii. (15 pts) If \( k = 3 \) and the mass is initially pulled 2 m to the right of the equilibrium and then imparted a 4 m/sec velocity to the left, will it pass through the equilibrium? If so, when? If not, briefly explain why not.

4. [APPM 2360 Exam (21 pts)] The following problems are not related.
   (a) (6 pts) Convert the initial value problem \( 3y''' + 9ty'' - 24\sqrt{t}y' - 30y = 60t - 36, \quad y(1) = 4, \quad y'(1) = -3, \quad y''(1) = 0 \) into a system of first order equations with appropriate initial condition. If possible, write this system as a matrix equation. If not possible, explain why not.
   (b) (15 pts) Use Laplace transform techniques to solve the initial value problem \( y'' - 4y' + 20y = 40, \quad y(0) = y'(0) = 0 \).