1. [APPM 2360 Exam (18 pts)] Let $L(y) = t^2 y'' - ty' + y$ with $t > 1$.

   (a) (6 pts) Show that the set $\{t, t \ln t\}$ forms a basis for the solution space of $L(y) = 0$.

   (b) (12 pts) Find the general solution of $L(y) = \dfrac{t}{\ln t}$. (Hint: u-substitution)

2. [APPM 2360 Exam (37 pts)] The following problems (a) and (b) are not related.

   (a) (22 pts) Consider the linear operator $L(y) = ay^{(4)} + by''' + cy'' + py' + qy$ where a, b, c, p, q are real constants.

      i. (4 pts) If the characteristic equation of $L(y) = 0$ has the root 1 of algebraic multiplicity 2 and the roots $2 \pm 3i$, write the general solution of $L(y) = 0$.

      ii. (18 pts) For $f(t)$ in each of the following, write the form (**DO NOT SOLVE FOR THE CONSTANTS**) of the particular solution that will be used for the method of undetermined coefficients when solving $L(y) = f(t)$. If the method of undetermined coefficients is not applicable, write N/A.

              A. $f(t) = 2t^{-1}$ B. $f(t) = e^t + \frac{t}{e^t}$ C. $f(t) = te^t + \sin t$

              D. $f(t) = \sin 4t + e^{2t} \cos 3t$ E. $f(t) = t^3 e^{t/2}$ F. $f(t) = \tan 3t + 2$

   (b) (15 pts) Use the method of undetermined coefficients to solve the initial value problem

   $$y''' - 2y'' = 4, \ y(0) = 9, \ y'(0) = 1, \ y''(0) = 14$$

3. [APPM 2360 Exam (24 pts)] The following problems (a) and (b) are not related.

   (a) (3 pts) Find the value(s) of b and m for which the harmonic oscillator governed by $m\ddot{x} + b\dot{x} + x = \cos(2\pi t)$ will have unbounded solutions.

   (b) (21 pts) A 3 kg mass is attached to a spring which in turn is attached to a wall. The entire apparatus is on a horizontal shelf and immersed in a fluid that offers a damping force equal to 6 times the instantaneous velocity. The oscillator is unforced and has a Spring-o-Matic dial marked with values of k that allows you to adjust the spring (restoring) constant, k.

      i. (6 pts) As the Spring-o-Matic dial is moved from $k = 5$ to 3 to 1, identify the type of damping (critical, over, under) that occurs for each of these values of k.

      ii. (15 pts) If $k = 3$ and the mass is initially pulled 2 m to the right of the equilibrium and then imparted a 4 m/sec velocity to the left, will it pass through the equilibrium? If so, when? If not, briefly explain why not.

4. [APPM 2360 Exam (21 pts)] The following problems are not related.

   (a) (6 pts) Convert the initial value problem $3y''' + 9ty'' - 24\sqrt{t}y' - 30y = 60t - 36$, $y(1) = 4$, $y'(1) = -3$, $y''(1) = 0$ into a system of first order equations with appropriate initial condition. If possible, write this system as a matrix equation. If not possible, explain why not.

   (b) (15 pts) Use Laplace transform techniques to solve the initial value problem $y'' - 4y' + 20y = 40$, $y(0) = y'(0) = 0$.