APPM 2360 **Exam 2** **Fall 2020**

1. [APPM 2360 Exam (24 pts)] Consider the following matrices:

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \qquad \mathbf{C} = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix}$$

Compute the following, if possible. If not possible, write "NOT DEFINED" and provide a brief explanation why.

(a) $\mathbf{AB}$ (b) $\mathbf{AC}$ (c) $\mathbf{A}^{\mathrm{T}}\mathbf{A}$ (d) $\left|\mathbf{C}^{-1}\right|$ (e) $\mathbf{B}^{\mathrm{T}}\mathbf{B}$ (f) $\mathbf{C} - \mathbf{C}^{\mathrm{T}}$ (g) $\left(\mathbf{BB}^{\mathrm{T}}\right)^{-1}$ (h) $\mathbf{CA}^{-1}$

2. [APPM 2360 (32 pts)] The following problems are not related. Be sure to provide thorough justification for your answers.

   (a) (8 pts) For what value(s) of t is the set of vectors $\left\{ \begin{bmatrix} t \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 4t \end{bmatrix} \right\}$ linearly independent?

   (b) (8 pts) Does the set of vectors $\left\{2 - x, 2x - x^2, 6 - 5x + x^2, x\right\}$ form a basis for $\mathbb{P}_2$?

   (c) (8 pts) Are the functions $\{t, \sin t, \cos t\}$ linearly independent on the real line?

   (d) (8 pts) Consider the vector space $\mathbb{M}_{22}$, the set of all 2×2 matrices with the standard operations for matrix addition and scalar multiplication.

      i. Let $\mathbb{W}_1$ be the set of matrices of the form $\begin{bmatrix} a & b \\ -b & c \end{bmatrix}$ where a, b, c are real numbers. Is $\mathbb{W}_1$ a subspace of $\mathbb{M}_{22}$?

      ii. Let $\mathbb{W}_2$ be the set of matrices of the form $\begin{bmatrix} a & 2 \\ -2 & b \end{bmatrix}$ where a, b are real numbers. Is $\mathbb{W}_2$ a subspace of $\mathbb{M}_{22}$?

3. [APPM 2360 Exam (20 pts)] The following problems are not related.

   (a) (5 pts) Is $\vec{\mathbf{v}} = \begin{bmatrix} 2 \\ -4 \\ 6 \end{bmatrix}$ an eigenvector of $\mathbf{A} = \begin{bmatrix} -1 & -1 & 1 \\ -2 & 0 & -2 \\ 3 & -3 & 1 \end{bmatrix}$? Hint: Use an appropriate definition.

   (b) (5 pts) Find the eigenvalues of $\mathbf{A} = \begin{bmatrix} 3 & 5 \\ -1 & -1 \end{bmatrix}$. Simplify your answer and **DO NOT** find the eigenvectors.

   (c) (10 pts) The characteristic equation for the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 & -2 \\ -2 & 5 & -2 \\ -6 & 6 & -3 \end{bmatrix}$ is $(\lambda - 3)^2(\lambda + 3) = 0$. Find a basis for and the dimension of the eigenspace corresponding to the repeated eigenvalue.

4. [APPM 2360 Exam (24 pts)] The following problems are not related.

   (a) (4 pts) For what value(s) of k can Cramer's Rule be used to solve the linear system

$$\begin{aligned} x + ky &= 10 \\ 7x + 21y &= -4 \end{aligned}$$

   (b) (8 pts) Use Gauss-Jordan elimination to find the inverse matrix and use the inverse to solve the system

$$\begin{aligned} x_1 + 2x_2 &= 6 \\ 3x_1 + 4x_2 &= 14 \end{aligned}$$

   No points for using a formula to find the inverse matrix.

   (c) (12 pts) Consider the linear system

$$\begin{aligned} x_1 - x_2 - x_3 &= -1 \\ 2x_1 + x_2 - 2x_3 &= -2 \\ -x_1 + 2x_2 + x_3 &= 1 \end{aligned}$$

      i. (8 pts) Use Gauss-Jordan elimination to transform the augmented matrix of the system to RREF.

      ii. (3 pts) Use the Nonhomogeneous Principle to write the solution of the linear system in the form $\vec{\mathbf{x}} = \vec{\mathbf{x}}_h + \vec{\mathbf{x}}_p$.

      iii. (1 pts) What is the dimension of the solution space of the associated homogeneous system?