1. [APPM 2360 Exam (20 pts)] The following problems are not related.
   (a) (6 pts) What conclusions can be drawn from Picard's Theorem regarding the existence and uniqueness of solutions to the initial value problem $y' = t/y,\ y(2) = 0$? Briefly explain.
   (b) (6 pts) With a step size of $h = 0.5$, use Euler's method to approximate the solution of the IVP $y' = t - y,\ y(1) = 2$ at $t = 2$.
   (c) (8 pts) Consider the following system of differential equations
   $$x' = 1 + x - y$$
   $$y' = -1 + x^2 + y^2$$
   i. (4 pts) Sketch and label the h and v nullclines in the phase plane.
   ii. (4 pts) Find all equilibrium points, if any exist.

2. [APPM 2360 (18 pts)] The following problems are not related.
   (a) (10 pts) Consider the differential equation $y' = y^2(y+4)^2(y^2-4)$.
   i. (4 pts) Find all equilibrium solutions and their stability.
   ii. (6 pts) Plot the phase line for the differential equation.
   (b) (8 pts) Given the differential equation $y' - y + 2t = 0$, draw the isoclines corresponding to slopes of $1, 0, -1$. Be sure to include the line segments showing the slope on each isocline.

3. [APPM 2360 Exam (22 pts)] Consider the initial value problem $y' = \dfrac{y^2 + 2ty}{t^2},\ y(1) = -2,\ t > 0$.
   (a) (2 pts) Letting $v = y/t$, show that $y' = tv' + v$.
   (b) (2 pts) Use part (a) to show that the original differential equation can be rewritten as $tv' = v^2 + v$.
   (c) (12 pts) Solve the differential equation in part (b).
   (d) (3 pts) Find the general solution to the original differential equation, writing your answer explicitly as $y(t) = \cdots$.
   (e) (3 pts) Find the solution to the original initial value problem.

4. [APPM 2360 Exam (16 pts)] Consider the differential equation $ty' - 2y = t^4 e^{t^2},\ t > 0$.
   (a) (2 pts) Show that $y_h(t) = Ct^2$ is a solution of the associated homogeneous equation. C is an arbitrary constant.
   (b) (12 pts) Use the Euler-Lagrange two-stage method (variation of parameters) to find a particular solution to the nonhomogeneous equation. Show all your work.
   (c) (2 pts) Use the Nonhomogeneous Principle to write the general solution to the nonhomogeneous differential equation.

5. [APPM 2360 Exam (24 pts)] The following problems are not related.
   (a) (10 pts) Suppose that a room containing 1000 ft^3 of air is originally free of a certain pollutant. Beginning at time $t = 0$ air, containing the pollutant at a concentration of 0.05 g/100 ft^3, is introduced into the room at a rate of 0.1 ft^3/min, and the well-circulated mixture is allowed to leave the room at the same rate. Let $x(t)$ be the amount of pollutant (grams) at time t. Set up, but **DO NOT SOLVE**, the initial value problem governing this situation. Simplify your answer.
   (b) (14 pts) Consider the differential equation $xy' + 2y = -\dfrac{\sin x}{x},\ x > 0$.
   i. (12 pts) Using the integrating factor method, find the solution that passes through the point $(\pi/2, 0)$.
   ii. (2 pts) For what value of x does $y = -1/\pi^2$?