# APPM 2360: Midterm Exam 3

November 28, 2018. 1.5 hours.

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ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your lecture section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **1 side only**) crib sheet is allowed. Each question is worth 20 points.

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**Problem 1 (True/False)** Are the following statements true or false? (Only answer TRUE or FALSE is needed; no justification is required.) If a statement is not always true, reply 'FALSE'. Give your answers in the bluebook (and not on this problem sheet).

(a) Observe that $y(t) = \cos(2t)$ is a solution to the ODE $y'' - y' + 4y = 2\sin(2t)$.
Is $y(t) = -\cos(2t)$ also a solution to this ODE?

(b) When using the method of undetermined coefficients for the ODE $y'' + y = 6\cos(t)$ the particular solution should be of the form $y_p(t) = A\cos(t) + B\sin(t)$.

(c) $-2\mathcal{L}\{te^{-t^2}\} = s\mathcal{L}\{e^{-t^2}\} - 1$, where $\mathcal{L}\{f(t)\}$ is the Laplace transform of $f(t)$.

(d) The functions $t^3$ and $\left|t^3\right|$ have the same Laplace transform.

(e) The Laplace transform $\mathcal{L}\{e^{t^3}\}$ exists for some $s$.

(f) All solutions to the equation $y'' + 4y = 0$ can be written in the form $y = A\sin(2t + \delta)$ for some real numbers $A$ and $\delta$.

(g) All solutions to the equation $y'' + 2y' + y = 0$, can be written in the form $y = ce^{-t}$ for some real number $c$.

(h) The equation $y''' + 2y'' + 4t^2y' - y = \cos(t)e^{-t^2}$ can be converted to 3 first-order linear ODEs in 3 dependent variables.

(i) The set of functions $\{\cos(t), \cos(t + 1), \cos(t + 2)\}$ is linearly independent.

(j) The set of functions $\{\cos(t), \cos(2t), \cos(3t)\}$ is linearly independent.

**Problem 2** The following questions are unrelated.

(a) Find a particular solution to

$$y'' - 4y' - 12y = 4e^{3t}$$

using the method of undetermined coefficients.

(b) Find the <u>full</u> general solution to

$$y'' + 4y = \frac{1}{\cos(2t)}.$$

<u>Hint:</u> $\int \tan(at)dt = -\frac{1}{a}\ln(\cos(at)) + C$

**Problem 3**

(a) Find a solution to the initial value problem $y'' + 4y' + 13y = 0, \;\; y(0) = 1, \; y'(0) = 10.$

(b) Find a basis for the space of solutions to the equation $y''' - y'' = 0$.

(c) Match each ODE with a graph that could correspond to a solution to the ODE

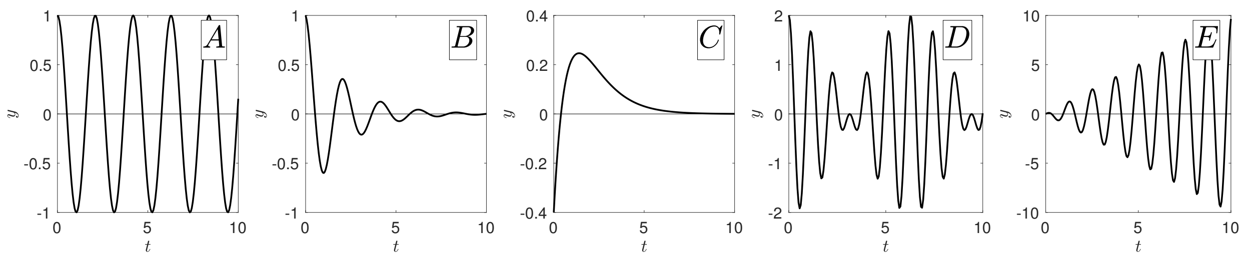
(i) $y'' + 2y' + y = 0$

(ii) $y'' + 25y = -10\sin(5t)$

(iii) $y'' + 9y = 0$

(iv) $y'' + 25y = 11\cos(6t)$

(v) $y'' + y' + 9.25y = 0$




**Problem 4** Solve the initial value problem

$$y'' - 5y' + 6y = 0$$

with $y(0) = 2$ and $y'(0) = 2$ by using Laplace transforms.

**Problem 5:**

(a) Find the Laplace transform of the function $f(t)$,

$$f(t) = \begin{cases} 2 & 0 \le t < 1, \\ 8e^{-t} & t \ge 1. \end{cases}$$

(b) Find the inverse Laplace transform of the function

$$F(s) = \frac{-7}{(s-1)^2 + 4} + \frac{8}{(s-1)^2 + 5}$$