APPM 2360: Midterm exam 2

October 24, 2018 1.5 hours

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **1 side only**) crib sheet is allowed.

Problem 1: (20 points) Are the following statements true or false? (Only answer TRUE or FALSE is needed; no justification is required.) If a statement is not always true, reply 'FALSE'. Give your answers in the bluebook (and not on this problem sheet).

(a) The vectors $\vec{\mathbf{v_1}} = (1, 0, 0, 1)$, $\vec{\mathbf{v_2}} = (0, 1, 0, 0)$ and $\vec{\mathbf{v_3}} = (0, 0, 1, 0)$ span $\mathbb{R}^4$.

(b) The dimension of the space of diagonal $n \times n$ real matrices with respect to the usual matrix addition and multiplication by real numbers is n^2.

(c) If $\{v_1, v_2, v_3\}$ is a basis of a vector space V, then v_1 and v_3 are linearly independent.

(d) If A is an $m \times n$ matrix and RREF(A) has some zero rows, then rank(A) is always less than n.

(e) Every nonzero $m \times n$ matrix has at least one pivot.

Problem 2: (20 points)

(a) Given n functions $f_1(x)$, $f_2(x)$, $\ldots$, $f_n(x)$, give the definition for these functions being linearly independent.

(b) Consider the specific case of the functions $\{x, 1 + x, 2 + 3x\}$. Determine whether these three functions are linearly independent, or not.

(c) Define the Wronskian $W(x)$ of a general set of n functions $f_1(x)$, $f_2(x)$, $\ldots$, $f_n(x)$.

(d) Apply the Wronskian to the set of functions listed in part (b) of this problem. What can you conclude from the resulting function $W(x)$ regarding linear independence?

Problem 3: (20 points) Consider the following matrices:

$$A = \begin{pmatrix} 0 & 0 & 2 \\ -1 & 1 & 2 \end{pmatrix} \qquad C = \begin{pmatrix} 1 & 0 & 0 \\ -4 & 3 & 0 \\ -4 & 2 & 1 \end{pmatrix}$$

$$B = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$$

Find the following quantities:

(a) $C + 2B$

(b) AB^T

(c) The eigenvalues of B and their multiplicities.

(d) The eigenvalues <u>and</u> eigenvectors of D.

Problem 4: (20 points)

(a) Find the inverse of the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & 6 \\ 0 & 0 & 1 \end{bmatrix}$

(b) Solve the following system of equations for x and y:

$$\begin{aligned} x + y &= 2 \\ 5x + 4y &= 3 \end{aligned}$$

Hint: observe $\begin{bmatrix} 1 & 1 \\ 5 & 4 \end{bmatrix} \cdot \begin{bmatrix} -4 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Problem 5: (20 points) This problem consists of three independent parts.

(a) Consider the set S of all functions $f(x)$ continuous on $[0, 1]$ and such that $f(0) = f(1)$. Is this a vector space? Justify your answer.

(b) Express the given vector $\vec{\mathbf{v}}$ in terms of the given basis $\{\vec{\mathbf{v_1}}, \vec{\mathbf{v_2}}\}$ of the vector space $\mathbb{R}^2$, where

$$\vec{\mathbf{v}} = \begin{pmatrix} a \\ b \end{pmatrix}, \qquad \vec{\mathbf{v_1}} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}, \quad \vec{\mathbf{v_2}} = \begin{pmatrix} -1 \\ 2 \end{pmatrix}.$$

(c) Consider $\mathcal{P}_4$, the vector space of all polynomials $p(x)$ of degree ≤ 4. Are $p_1(x) = x^3 - 3x + 1$, $p_2(x) = x^4 - 6x + 3$ and $p_3(x) = x^4 - 2x^3 + 1$ linearly independent elements of $\mathcal{P}_4$? What is the dimension of the subspace of $\mathcal{P}_4$ they span? Give an example of a basis for this subspace. Justify your answers.