APPM 2360: Midterm Exam 1
September 26, 2018. 1.5 hours.

ON THE FRONT OF YOUR BLUEBOOK write: (1) your name, (2) your instructor's name, (3) your recitation section number and (4) a grading table. Text books, class notes, cell phones and calculators are NOT permitted. A one page (letter sized **1 side only**) crib sheet is allowed.

Problem 1 (18 pts): Let $m(t)$ be the amount of mass in kg of a radioactive substance, and assume dm/dt is proportional to m due to radioactive decay. The amount of substance is measured at 3 kg at $t = 0$. One minute later the substance is measured at 2 kg.

(a) (10 pts) How much substance remains after another minute has passed (i.e., two minutes into the experiment)?

(b) (8 pts) Use Euler's method with step size $h = 1$ to approximate the remaining amount of radioactive substance at $t = 1$ and $t = 2$ starting from $t = 0$. There is no need to approximate logarithms.

Problem 2 (20 pts): For each of the five differential equations in the rows of the table below, fill out the table by answering the questions. The region R is $R = \{(t, y) \mid -4 < t < 4, \ -4 < y < 4\}$. The last two rows are asking about solutions that satisfy the initial condition $y(t_0) = y_0$. **Please copy the table and fill it out in your blue book.** Some cells may be left blank if appropriate. No justification is needed for this problem.

	$y' = y^{1/3}t$	$y' = -y + t$	$ty' + y = 0$	$ty' + y = 2$	$y' = \sqrt{\lvert y \rvert}$
Matching direction field from Fig. 1					
Is the equation linear?					
If linear, is it homogeneous?					
If $y_1(t)$ and $y_2(t)$ both solve the ODE, does $y_3(t) = y_1(t) + 7y_2(t)$ also solve the ODE?					
Is the equation separable?					
Can you guarantee a solution exists for all (t_0, y_0) in R?					
Can you guarantee solutions are unique for all (t_0, y_0) in R?					

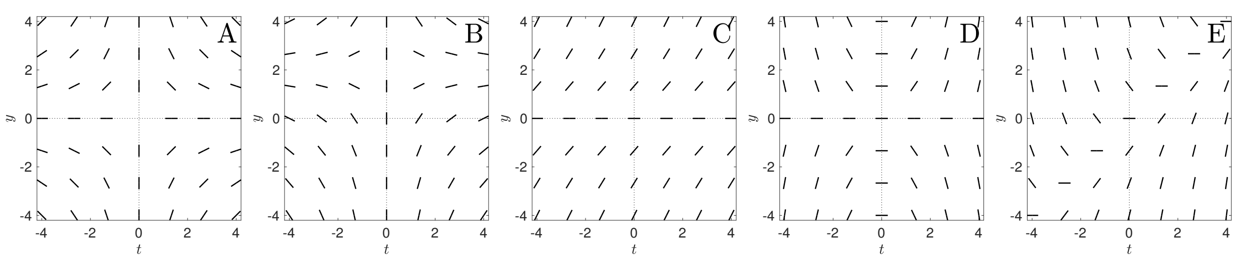


Figure 1: Direction fields for use in problem 2

Problem 3 (16 pts): The follow questions are unrelated. Please answer each question with a short response. Justify your answers.

(a) (8 pts) Short answer: Draw the phase line corresponding to the ODE $y' = (y-2)(y+3)y^2$. If you find any, characterize the equilibrium solutions: give their position and stability. Without computing the solution but using the phase line to justify your answer, determine the long term behavior of the solution to the equation with the initial value $(t_0, y_0) = (-4, 1)$.

(b) (8 pts) Short answer: Consider the IVP for the logistic equation $\frac{dy}{dt} = 5(1-y)y$ with $y(0) = y_0$. Describe the behavior of the solution (including, but not limited to, long term behavior) when

(i) $0 < y_0 < 1$

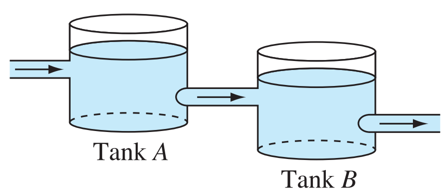
(ii) $y_0 > 1$

Problem 4 (22 pts): Consider an internally uniform spherical shell with inner radius 1m and outer radius 2m. The inner surface is kept at a constant 15°C, while the outer surface is kept at a constant 25°C. The long-term temperature distribution $T(r)$ within the shell ($1 \le r \le 2$) satisfies

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0.$$

Solve this to get an expression for the temperature $T(r)$. Your answer should not involve any unknown constants. Hint: Let $S = \frac{dT}{dr}$.

Problem 5 (24 pts)



Consider two tanks T_A and T_B, each holding 10 liters of water initially with 2 kg and 5 kg of dissolved salt respectively. We want to flush the tanks clean with pure water. Assume the tanks always stay well mixed. The amount of salt in each tank is defined as $A(t)$ and $B(t)$. Let 1 L/ min of pure water enter T_A, the same amount then flows from T_A into T_B, and finally the same amount leaves T_B. The governing equations for $A(t)$ and $B(t)$ are

$$\frac{dA}{dt} = -\frac{A}{10}$$

$$\frac{dB}{dt} = \frac{1}{10}(A - B).$$

(a) (12 pts) Solve for $A(t)$ and $B(t)$.

(b) (6 pts) Describe in general terms what the solutions, $A(t)$ and $B(t)$, look like (for example what are their initial slopes and long term behavior?).

(c) (6 pts) A pump is added to T_B which pumps 1 L/ min into T_A. To keep the water levels the same, the flow rate from T_A into T_B is increased to 2 L/ min. Write down the governing differential equations (you do not need to solve). Sketch the nullclines (and directional arrows), and label any equilibrium points and the initial condition.