- This exam is worth 150 points and has 6 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"$\times$ 11" crib sheet with writing on both sides.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/072624 (24 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The function $f(x,y) = 5x + y^{1/3}$ has no critical points.

   (b) $x^2 - 2y^2 + 3z^2 + 4x = -1$ is a hyperboloid of two sheets.

   (c) The planes $x + y - 3z = 4$ and $-5x - 5y = -15z$ do not intersect.

   (d) If you are standing on the surface $f(x,y) = 1 - x^2 - y^3$ at the point $(1,-1,1)$ and begin walking in the direction $6\,\mathbf{i} - 4\,\mathbf{j}$, you will be following a level curve of $f(x,y)$.

   (e) The acceleration vector of the curve $\mathbf{r}(t) = -2\sin t\,\mathbf{j} - 2\cos t\,\mathbf{k}$ lies in the xz-plane.

   (f) The symmetric equations of the line through the origin in the direction of $\langle 1,1,1\rangle$ are $x = y = z$.

   (g) The vector field $\mathbf{V}(x,y,z) = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$ is both irrotational and incompressible.

   (h) For all $(x_0,y_0) \in \mathbb{R}^2$, $\displaystyle\lim_{(x,y)\to(x_0,y_0)} \frac{\cos x + \sin y}{1 + x^2 + y^2} = \frac{\cos x_0 + \sin y_0}{1 + x_0^2 + y_0^2}$.

2. [2350/072624 (24 pts)] Let $f(x,y) = \tan^{-1}\dfrac{y}{x}$ and consider the path, $\mathcal{C}$, given by $\mathbf{r}(t) = \sqrt{2}\sin t\,\mathbf{i} + \sqrt{2}\cos t\,\mathbf{j}$, $\dfrac{\pi}{4} \le t \le \dfrac{3\pi}{4}$. Recall that $\left(\tan^{-1} x\right)' = (1+x^2)^{-1}$.

   (a) (10 pts) Show that $f(x,y)$ is a potential function for $\mathbf{E} = -\dfrac{y}{x^2+y^2}\,\mathbf{i} + \dfrac{x}{x^2+y^2}\,\mathbf{j}$.

   (b) (10 pts) Find the work done moving an object through the vector field along the given path.

   (c) (4 pts) Find the work done moving an object around any ellipse located completely in the fourth quadrant.

3. [2350/072624 (48 pts)] Consider a metal plate consisting of the first octant portion of the cylinder $x - y = 0$, $0 \le z \le 2$, $0 \le x \le 1$. The plate is made of a metal alloy whose density is $\delta(x,y,z) = x + y + 1$.

   (a) (16 pts) Find the mass of the plate.

   (b) (16 pts) Using a normal vector to the plate oriented with a positive **i**- and negative **j**-component, show that the flux of $\mathbf{F} = (x+y)\,\mathbf{i} - \mathbf{j} + z\,\mathbf{k}$ through the plate equals the mass of the plate divided by $\sqrt{2}$. Hint: take advantage of the work you did in part (a).

   (c) (16 pts) Suppose the plate represents a wall on a building. The height of the roof of the building is $z = 1 + x + y$ and the wall extends from the xy-plane up to the roof. Use an appropriate integral to find the area of one side of the wall.

MORE PROBLEMS BELOW/ON REVERSE

4. [2350/072624 (16 pts)] Use Stokes' theorem to evaluate $\displaystyle\iint_{\mathcal{S}} \nabla \times \mathbf{V} \cdot \mathbf{n}\, \mathrm{d}S$ where $\mathcal{S}$ is given by $x^2 + y^2 - z^2 = -1,\ -\sqrt{5} \le z \le -1$, upward pointing normal, and $\mathbf{V} = (y - x)\,\mathbf{i} - (x + y)\,\mathbf{j} + e^{xyz}\,\mathbf{k}$.

5. [2350/072624 (22 pts)] Consider the solid region, $\mathcal{W}$, enclosed below the sphere $x^2 + y^2 + z^2 = 2$ and above the plane $z = 1$ and let $\partial\mathcal{W}$ be its boundary.

   (a) (16 pts) Use Gauss' Divergence theorem to find the flux of $\mathbf{F} = e^{yz}\,\mathbf{i} + xz\,\mathbf{j} + 2(z^2 - 1)\,\mathbf{k}$ through $\partial\mathcal{W}$.

   (b) (6 pts) Briefly explain in words why the flux in part (a) is the same as the flux through just the spherical portion of $\partial\mathcal{W}$.

6. [2350/072624 (16 pts)] Use Green's theorem to find the circulation of $\mathbf{F} = 3xy\,\mathbf{i} + y^2\,\mathbf{j}$ around the semicircle $x^2 + y^2 = 9,\ x \le 0$, oriented counterclockwise.