- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

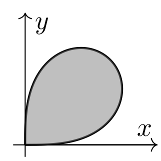
0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/071224 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) The integrals $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}}\left(1-x^2-y^2\right)\,\mathrm{d}y\,\mathrm{d}x$ and $\displaystyle\int_0^1\int_0^{\sqrt{1-y^2}}\int_0^{1-x^2-y^2}\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$ compute the same quantity.

   (b) For all $f(x,y)$ that are continuous on $\mathbb{R}^2$, $\displaystyle\int_0^1\int_{-y}^{y} f(x,y)\,\mathrm{d}x\,\mathrm{d}y = \int_{-1}^{0}\int_0^{-x} f(x,y)\,\mathrm{d}y\,\mathrm{d}x + \int_0^1\int_0^x f(x,y)\,\mathrm{d}y\,\mathrm{d}x$.

   (c) The area of the following figure is given by $\displaystyle\int_0^{2\pi}\int_0^{\sqrt{\sin 2\theta}}\mathrm{d}r\,\mathrm{d}\theta$.

   (d) $(x,y,z) = (-2,2,-2\sqrt{2})$ and $(\rho,\theta,\phi) = \left(4,\dfrac{7\pi}{4},\dfrac{3\pi}{4}\right)$ represent the same point in $\mathbb{R}^3$.

   (e) $\rho = -2\cos\phi$ is a sphere of radius 1 centered at $(0,0,-1)$.

2. [2350/071224 (20 pts)] After the first bite, a slice of pizza occupies the region, $\mathcal{D}$, in Quadrant II, bounded by the lines $y=-x$ and $x=0$ and the arc of the circles $x^2+y^2=9$ and $x^2+y^2=1$. The density of the pizza is $\delta(x,y) = \dfrac{6x^2}{x^2+y^2}$. If the mass of the pizza slice is $3(\pi-2)$, find $\bar{y}$, the y-coordinate of its center of mass.

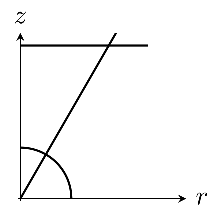
3. [2350/071224 (23 pts)] Use the change of variables $u=x-y,\ v=x+y$ to evaluate $\displaystyle\int_1^3\int_1^{4-x}\left(\frac{x-y}{x+y}\right)\mathrm{d}y\,\mathrm{d}x$.

MORE PROBLEMS BELOW/ON REVERSE

4. [2350/071224 (29 pts)] Let $\mathcal{E}$ be the first octant portion of the solid region that consists of all points satisfying

$$x^2 + y^2 + z^2 \geq 4 \quad \text{AND} \quad z \leq 6 \quad \text{AND} \quad z \geq \sqrt{3x^2 + 3y^2}$$

A helpful figure of an arbitrary θ plane is shown below. It is drawn to scale, but you will need to find the specific intersection locations and correctly identify the region of integration based on the three inequalities shown above.



The temperature in the solid is given by $T(x, y, z) = z\left(x^2 + y^2\right)$. Fully set up, but **DO NOT EVALUATE**, integral(s) to find:

(a) (15 pts) The *average temperature* of the region using spherical coordinates and integration order $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$.

(b) (14 pts) The *volume* of the region using cylindrical coordinates and integration order $\mathrm{d}r\,\mathrm{d}z\,\mathrm{d}\theta$

5. [2350/071224 (18 pts)] Evaluate $\displaystyle\int_0^1 \int_{\sqrt[3]{z}}^1 \int_0^{\ln 3} \frac{\pi e^{2x} \sin(\pi y^2)}{y^2}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$. Hint: $\dfrac{\sin y^2}{y^2}$ has no elementary antiderivative.