- This exam is worth 100 points and has 5 problems.
- Show all work and simplify your answers! Answers with no justification will receive no points unless otherwise noted.
- Begin each problem on a new page.
- **DO NOT LEAVE THE EXAM UNTIL YOUR HAVE SATISFACTORILY SCANNED AND UPLOADED YOUR EXAM TO GRADESCOPE.**
- You are taking this exam in a proctored and honor code enforced environment. No calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"× 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting. Remote students cannot interact with anyone except the proctor during the exam.

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0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/062824 (10 pts)] Write the word **TRUE** or **FALSE** as appropriate. No work need be shown. No partial credit given.

   (a) $f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$ is continuous on $\mathbb{R}^2$.

   (b) The level curves of the function $z = 8x^2 - 5y^2$ for all real values of k are parabolas.

   (c) Given $f(u(x,y,z), v(x,y,z))$, the rate of change of f with respect to x is $f_x x_u + f_y y_u$.

   (d) The tangent plane to the surface $e^x + e^y + e^z = 3$ at $(0,0,0)$ is $x + y + z = 0$

   (e) Suppose you move in a direction of the unit vector $\mathbf{u} = \langle u_1, u_2 \rangle$ that makes an angle of $\pi/3$ with the nonzero gradient vector of $f(x,y)$ at a point (x_0, y_0). Then the rate of change of f with respect to distance will be half of the maximum rate of increase of f at that point.

2. [2350/062824 (23 pts)] Consider the function $f(x,y) = x^3 + y^3 + x^2y^2$

   (a) (10 pts) Calculate the first order Taylor approximation to $f(x,y)$ centered at the point $(1,1)$.

   (b) (4 pts) Use your result from part (a) to estimate the value of $f(0.9, 1.1)$.

   (c) (9 pts) Now suppose you actually worked out the second order Taylor approximation to $f(x,y)$ centered at $(1,1)$ (you do not actually need to do it). Calculate an upper bound on the absolute value of the error associated with the second order approximation assuming that you only use values of x and y such that $|x-1| \leq 0.1$ and $|y-1| \leq 0.2$.

3. [2350/062824 (16 pts)] Andrea the ant is crawling around the ground along the path given by $x^2 + y^2 = 2$. The temperature of the ground where she is walking is $T(x,y) = (x-1)^2 + (y-1)^2 + 70$. Use Lagrange multipliers to answer the following questions.

   (a) If Andrea wants to stay away from the warmest spot(s) on the ground, which point(s) should she avoid? What is the temperature of the warmest spot(s)?

   (b) If she wants to chill in the coldest spot(s), where should she go and what will the temperature be there?

MORE PROBLEMS BELOW/ON REVERSE

4. [2350/062824 (25 pts)] A charged metal plate occupies the square region of the xy-plane given by $|x| \leq 3$, $|y| \leq 3$. The charge on the plate is $q(x, y) = 6y^2 - 2y^3 + 3x^2 + 6xy$.

   (a) (5 pts) Find the critical points of the charge function.

   (b) (5 pts) Are there any points, P, on the plate where the charge increases in some directions from P and decreases in others? If so, find them and the value of the charge there. If not, explain why not.

   (c) (5 pts) Are there any points, Q, on the plate where the charge decreases in all directions from Q? If so, find them and the value of the charge there. If not, explain why not.

   (d) (5 pts) Are there any points, R, on the plate where the charge increases in all directions from R? If so, find them and the value of the charge there. If not, explain why not.

   (e) (5 pts) Will the plate possess an absolute maximum and minimum charge? Justify your answer. (If the extrema exist, you do not need to find them.)

5. [2350/062824 (26 pts)] The following problems are not related. Justify your answer to all parts using mathematical techniques learned in this Calculus III course.

   (a) (8 pts) The mass of a certain object is given by the function $m = \frac{2}{3}\pi l^3 w^{\frac{3}{2}}$ where l is the length of the object and w is its width. Consider two such objects, one (I) with a length of 1 unit and a width of 4 units and another (II) with a length of 4 units and a width of 1 unit. Which object's mass is more sensitive to a small change in length?

   (b) (18 pts) A mosquito is buzzing along the path $\mathbf{r}(t) = \frac{t^3}{12}\,\mathbf{i} + \frac{t^2}{2}\,\mathbf{j} + t\,\mathbf{k}$. Length units are meters and time units are minutes. The mosquito is flying in an area where the carbon dioxide concentration, $C(x, y, z)$, in parts per million (ppm), an attractant for mosquitoes, is given by $C(x, y, z) = 3xyz$. Answer the following questions when the mosquito's path passes through the point $\left(\frac{2}{3}, 2, 2\right)$, making sure to include appropriate units.

      i. (6 pts) What is the instantaneous rate of change of the carbon dioxide concentration with respect to time?

      ii. (6 pts) What is the instantaneous rate of change of the carbon dioxide concentration with respect to distance (arc length)?

      iii. (6 pts) What is the greatest rate of change of the carbon dioxide concentration with respect to distance that the mosquito could observe?