- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. $\boxed{\textbf{NO}}$ calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5"$\times$ 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

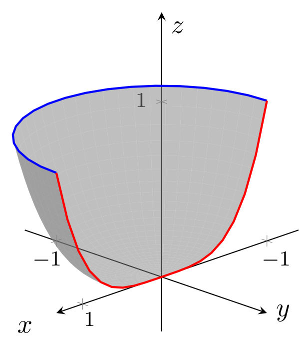
1. [2350/071423 (20 pts)] The portion of river, $\mathcal{D}$, that Penelope the platypus inhabits is bounded by the functions $x+y+4=0$, $x+y+4=8$, $x-2y=-1$, and $x-2y=1$. The depth (meters) of Penelope's river is described by $g(x,y)=3y+4$. Use a change of variables to find the volume of Penelope's river in the region $\mathcal{D}$. Include appropriate units in your answer.

2. [2350/071423 (27 pts)] During a tea party with the Mad Hatter, you are given exactly a 1/2 teacup of tea (pictured below) where the surface of the teacup is given by the equation $z=(x^2+y^2)^2$. It lies in the region where $y \le 0$ and $0 \le z \le 1$. You wish to know the volume of tea you received. For each part below, set up the triple integral using the given coordinate system and order of integration. **DO NOT EVALUATE** the triple integrals. Note: To receive full credit your bounds **must** match the solid as it is pictured (so study the figure closely).

   (a) Cartesian Coordinates with order $\mathrm{d}y\,\mathrm{d}z\,\mathrm{d}x$.

   (b) Cylindrical Coordinates with order $\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

   (c) Spherical coordinates with order $\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$



3. [2350/071423 (18 pts)] Rewrite the following integrals as a single integral in polar coordinates **and** evaluate it. Drawing a picture should be beneficial.

$$\int_{-2}^{-1}\int_{-x}^{\sqrt{8-x^2}} 3x\,\mathrm{d}y\,\mathrm{d}x + \int_{-1}^{0}\int_{\sqrt{2-x^2}}^{\sqrt{8-x^2}} 3x\,\mathrm{d}y\,\mathrm{d}x$$

4. [2350/071423 (22 pts)] Consider a thin leaf whose shape can be described by the region bounded by the curves $y=x^2$ and $y=x^4$ with $0 \le x \le 1$.

   (a) (16 pts)] Assume the mass of the leaf is $\frac{1}{20}$ grams and its mass density is $\rho(x,y)=x\sqrt{y}$ grams per square centimeter.

      i. (6 pts)] Find the moment of the leaf about the y-axis using the integration order $\underline{\mathrm{d}y\,\mathrm{d}x}$.

      ii. (6 pts)] Find the moment of the leaf about the x-axis using the integration order $\underline{\mathrm{d}x\,\mathrm{d}y}$.

      iii. (4 pts)] What is center of mass, $(\bar{x},\bar{y})$, of the leaf?

   (b) (6 pts)] Now suppose the power density of sunlight hitting the leaf is $P(x,y)=k(1-x^2)$ watts per square centimeter and you know that the total power hitting the leaf is 8 watts. What is k?

5. [2350/071423 (13 pts)] Use spherical coordinates to evaluate $\displaystyle\iiint_{\mathcal{E}} z\,\mathrm{d}V$ where $\mathcal{E}$ is the solid that lies within the sphere $x^2+y^2+z^2=4$, above the xy-plane, below the cone $z=\sqrt{x^2+y^2}$ and between the planes $y=\dfrac{\sqrt{3}}{3}x$ and $y=\sqrt{3}x$.