

- This exam is worth 100 points and has 5 problems.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted.
- **Please begin each problem on a new page.**
- **DO NOT** leave the exam until you have satisfactorily scanned and uploaded your exam to Gradescope.
- You are taking this exam in a proctored and honor code enforced environment. **NO** calculators, cell phones, or other electronic devices or the internet are permitted during the exam. You are allowed one 8.5" × 11" crib sheet with writing on one side.
- Remote students are allowed use of a computer during the exam only for a live video of their hands and face and to view the exam in the Zoom meeting.

0. At the top of the first page that you will be scanning and uploading to Gradescope, write the following statement and sign your name to it: "I will abide by the CU Boulder Honor Code on this exam." **FAILURE TO INCLUDE THIS STATEMENT AND YOUR SIGNATURE MAY RESULT IN A PENALTY.**

1. [2350/061623 (45 pts)] Penelope the platypus leaves her burrow at $t = 0$ seconds and swims in the river in search of food, following the path given by (position in meters)

$$\mathbf{r}(t) = [\sin(t) - 2]\mathbf{i} + t\mathbf{j} + [\cos(t) - 1]\mathbf{k}$$

- (a) [8 pts] How fast is Penelope swimming?
- (b) i. [3 pts] What is Penelope's acceleration, $\mathbf{a}(t)$?
 ii. [4 pts] What are the tangential and normal components of Penelope's acceleration?
 iii. [3 pts] What is the unit normal to Penelope's path, $\mathbf{N}(t)$?
- (c) [5 pts] How much distance has Penelope traveled along the path when $t = \frac{5\pi}{3}$?
- (d) [10 pts] Penelope catches a crawfish at $t = \frac{5\pi}{3}$. She decides to take the crawfish to the surface of the river (described by the plane $z = 0$) to enjoy the snack. To get there, she swims in a straight line (tangent to the original path) at a speed of one meter per second. Find the arc length parameterization (think unit vector) of her straight line trajectory to the river's surface. Be sure to include a range for the parameter.
- (e) [6 pts] Using your answer for part (d), find where Penelope reaches the surface of the river ($z = 0$). How long did it take her to get there after catching the crawfish? Do not simplify your answer.
- (f) [6 pts] Using the position you found in part (e), how far is Penelope from her burrow? Do not simplify your answer.

2. [2350/061623 (12 pts)] Consider the quadric surface

$$y^2 - 6y + z^2 - x^2 + 2x = -4$$

- (a) [7 pts] Classify the quadric surface, describe how it is oriented and where it is in space.
- (b) [5 pts] Sketch and describe the trace when $x = 1$. Label your sketch appropriately.
3. [2350/061623 (15 pts)] Suppose the acceleration experienced by a particle moving along a path in space is $\mathbf{a}(t) = \langle 2, 0, 24t^2 \rangle$. Furthermore, suppose that the particle's velocity at time $t = 0$ is \mathbf{j} and that the particle is at the point $(3, 7, 2)$ when $t = -1$.
- (a) [10 pts] Where is the particle when $t = 1$?
- (b) [5 pts] Is the particle's acceleration vector ever orthogonal to its velocity vector? If so, when? If not, justify why not.

4. [2350/061623 (20 pts)] Consider the lines

$$L_1 : \frac{x+3}{4} = \frac{z+1}{2}, y = 4 \quad \text{and} \quad L_2 : \frac{x-1}{2} = y - 4 = \frac{z-1}{2}$$

- (a) [10 pts] Show that the lines are neither parallel nor skew. Justify your answer.
- (b) [10 pts] Find an equation of the plane containing these lines. Write your answer in standard form $ax + by + cz = d$.
5. [2350/061623 (8 pts)] $100\sqrt{3}$ ft-lb of work are done in moving a crate 10 feet along a horizontal boat dock. The constant force, \mathbf{F} , used to move the crate is applied at an angle of 30° above the horizontal. If this same force produces a torque of magnitude 5 ft-lb when applied at the same angle to the end of a wrench to loosen a bolt, how long is the wrench?