APPM 2350 **Final Exam** **Summer 2022**

This exam has 5 problems. Show all your work and simplify your answers. Answers with missing or insufficient justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

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1. (32 pts) Suppose the density of the surface $z = 1 - x^2$ is $\delta = |x|y$ g/cm^2 and consider the vector field

$$\mathbf{F} = \langle 3x + \cos y, 2y + \sin z, e^x + 5z \rangle$$

   (a) Find the mass of the part of the surface lying above the region in the xy-plane between $y = 0$ and $y = 2$.

   (b) Find the outward flux of $\mathbf{F}$ through the **closed** surface enclosing the region below $z = 1 - x^2$, above the xy-plane and between $y = 0$ and $y = 2$.

2. (16 pts) Find the area under the graph of $z = 100(x^2 + 2y^2)$ lying above the second quadrant portion of the **curve** $x^2 + y^2 = 4$.

3. (16 pts) I am doing laps around the unit circle (counterclockwise) in the presence of the force field

$$\mathbf{F} = \left\langle Axy - By^3, 4y + 3x^2 - 3xy^2 \right\rangle$$

   (a) After having gone from $(1, 0)$ to $(0, 1)$, I am already getting tired from all of the work I've done. A friend standing nearby tells me to chill because when I get back to $(1, 0)$ I will have done no work at all. What are A and B? Briefly explain.

   (b) If I go around the circle too much, I'll get dizzy so my friend tells me to go from $(-1, 0)$ to $(3, -2)$ along the path $y = \sqrt{x+1}(x-2)^{300}(x-4)^{301}$ instead. How much work will I do walking on that path?

4. (20 pts) Find the circulation of $\mathbf{V}(x, y, z) = \mathbf{i} + (x + yz)\,\mathbf{j} + \left(xy - \cos^2\sqrt{z}\right)\mathbf{k}$ around the closed path consisting of the straight line segments connecting the points $(1, 0, 0), (0, 0, 2), (0, 2, 0)$ and $(1, 0, 0)$, in that order, by completely setting up an appropriate **surface** integral. Do not evaluate your integral, but your final answer should include a fully simplified integrand, correct bounds, *etc.*

5. (16 pts) Use Green's Theorem to evaluate $\displaystyle\oint_{\mathcal{C}} -y^3 \mathrm{d}x + \left(x^3 + \sqrt{y^3+1}\right) \mathrm{d}y$. $\mathcal{C}$ is the closed counterclockwise path consisting of the bottom half of the unit circle and the portion of the x-axis with $-1 \leq x \leq 1$.

END OF EXAM
