APPM 2350—Exam 1

Friday, June 10th 1-2:35pm 2022

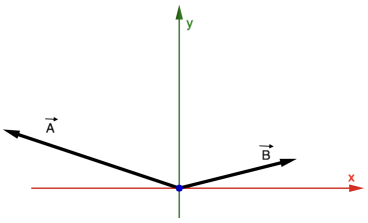
This exam has 4 problems. Show all your work and simplify your answers. Answers with no justification will receive no points. You are allowed one 8.5×11-in page of notes (ONE side). You may NOT use a calculator, smartphone, smartwatch, the Internet or any other electronic device.

Question 1 (20 pts)

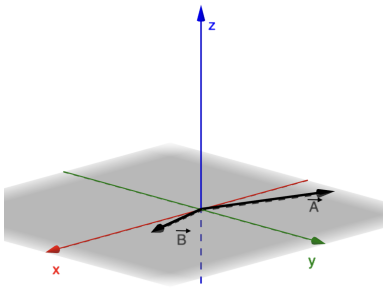
The following parts are not related:

(a) The vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, shown below, are parallel to the xy-plane:

2D view



3D view of the same vectors



On your own sheet of paper, sketch and clearly label diagrams for the following (do not draw on this exam sheet). Clearly label each vector and each axis.

(i) $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{B}} - \vec{\mathbf{A}}$

(ii) $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\mathbf{proj}_{\vec{\mathbf{B}}}\vec{\mathbf{A}}$.

(iii) $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$

(b) Find one possible force vector $\vec{\mathbf{F}}$ that satisfies both of the following criteria:

- The work done by $\vec{\mathbf{F}}$ in moving an object from the point $(2, -2, 4)$ to the point $(-2, 5, 6)$ is 12 N-m (where distances are measured in meters and force is measured in Newtons)
- $\vec{\mathbf{F}}$ is not parallel to the object's displacement

Question 2 (32 pts)

A drone travels along the path given by

$$\vec{\mathbf{r}}(t) = (t+1)\hat{\mathbf{i}} + 2t\hat{\mathbf{j}} + (2 + 3t - t^2)\hat{\mathbf{k}}$$

(a) Are there any time(s), t, such that the drone's velocity is orthogonal to its acceleration? If so, find these time(s). If not explain why.

(b) The drone's entire trajectory actually lies in one plane. Find the equation of that plane. (Hint: You do not need to find $\hat{\mathbf{T}}$ and $\hat{\mathbf{N}}$ to do this problem).

(c) At time $t = 2$ the drone fires a missile straight ahead along a line.

(i) How far from the point $(1, 1, 1)$ is the drone at the moment it fires the missile?

(ii) Give a vector-valued function that traces out the missile's path.

(iii) At what point(s) (x, y, z) does the missile intersect the surface $z + 5 = (y - 9)^2 - (x - 5)^2$?

CONT'D ON REVERSE

Question 3 (25 pts)
Consider a particle moving along a path $\vec{\mathbf{r}}(t)$ with a **constant speed** of 3 along the entire path. We also know that at the particular time $t = 5$ the following is true:

$$\vec{\mathbf{r}}(5) = 3\hat{\mathbf{j}} - 2\hat{\mathbf{k}} \qquad \hat{\mathbf{T}}(5) = \frac{\hat{\mathbf{i}} + 2\hat{\mathbf{k}}}{\sqrt{5}} \qquad \hat{\mathbf{N}}(5) = -\hat{\mathbf{j}} \qquad \kappa(5) = 2$$

For each of the following quantities, determine if you have enough information to calculate the exact quantity. If so, calculate it and justify your answer. If not, explain what additional information you'd need to calculate the exact quantity.

(a) $\hat{\mathbf{B}}(5)$
(b) $\hat{\mathbf{B}}(0)$
(c) $\vec{\mathbf{v}}(5)$
(d) $\vec{\mathbf{a}}(5)$
(e) $\vec{\mathbf{v}} \cdot \vec{\mathbf{a}}$ at the time $t = 10$

Question 4 (23 pts)
As part of an engineering project, you are trying to weld two steel objects together. The surfaces of the two objects are given by

$$\frac{(x-1)^2}{50} + \frac{y^2}{100} = 1 \quad \textit{and} \quad z = \frac{x^2}{2} + \frac{y^2}{4}$$

where distances are measured in feet. The objects are joined where these surfaces intersect. You know that to weld the objects together, you will need 0.05 pounds of welding wire per foot of weld.

(a) Classify (i.e. give the name of) both of the surfaces.
(b) Give a parameterization that traces out the curve of intersection of the surfaces. Give a parametric interval such that this curve is traced once.
(c) If you have 3 lbs of welding wire available, will you be able to complete the weld? Show work fully justifying your answer.

End Of Exam
