1. [2350/072321 Exam (45 pts)] Let $\mathbf{F} = y\,\mathbf{i} + yz\,\mathbf{j} - \frac{1}{2}x^2\,\mathbf{k}$ and consider the surface, $\mathcal{S}$, given by $x^2 + y^2 - z^2 = -1, 1 \le z \le \sqrt{5}$ with upward pointing normal.

   (a) (5 pts) Name the surface.

   (b) (20 pts) Find the circulation of $\mathbf{F}$ on the boundary of $\mathcal{S}$ by direct computation. The identity $1 - 2\sin^2 t = \cos 2t$ may be helpful.

   (c) (20 pts) Find the circulation of $\mathbf{F}$ on the boundary of $\mathcal{S}$ using Stokes' Theorem.

2. [2350/072321 Exam (25 pts)] Let $\mathbf{F} = e^y\,\mathbf{i} + (xe^y + \sin z)\,\mathbf{j} + y\cos z\,\mathbf{k}$.

   (a) (12 pts) Show that $\mathbf{F}$ is conservative.

   (b) (13 pts) Find the work done by $\mathbf{F}$ on an object that moves from $(0, 0, 0)$ to $(1, -1, 3)$.

3. [2350/072321 Exam (20 pts)] Use Green's theorem to find the outward flux of the vector field $\mathbf{F} = x^2y\,\mathbf{i} + 3xy^2\,\mathbf{j}$ through the boundary of the second quadrant portion of the circle of radius 3 centered at the origin.

4. [2350/072321 Exam (20 pts)] Let $\mathcal{S}$ be the first octant portion of the plane with intercepts $(2, 0, 0)$, $(0, 4, 0)$ and $(0, 0, 1)$. Its area is $\sqrt{21}$. Using this information, find the average value of $f(x, y, z) = 1 + x$ over $\mathcal{S}$.

5. [2350/072321 Exam (20 pts)] A piece of wire is in the shape of (e^t, t^2) with its left end at the point $(1, 0)$. The charge density on the wire is $q(x, y) = \dfrac{3y}{\sqrt{x^2 + 4y}}$ Coulombs per meter. If the total charge on the wire is 8 Coulombs, find the coordinates of the right end of the wire.

6. [2350/072321 Exam (20 pts)] Use Gauss' Theorem to find the outward flux of the vector field $\mathbf{F} = x^3\,\mathbf{i} + y^3\,\mathbf{j} + z^3\,\mathbf{k}$ through the boundary of the solid region above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = 4$.