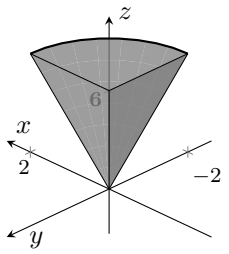
1. [2350/070921 Exam (15 pts)] Let $\mathcal{R}$ be the parallelogram in the xy-plane enclosed by the lines $x+4y=4, x+4y=9, x-y=1$ and $x-y=4$. Use a change of variables to find the volume of the solid above the region $\mathcal{R}$ and below the surface $z=5\sqrt{(x-y)(x+4y)}$.

2. [2350/070921 Exam (15 pts)] A metal pipe with inner diameter d_i, outer diameter d_o and length l has a mass density that varies inversely with the cube of the distance from the axis of the pipe, that is, mass density, $\delta = k/\text{distance}^3$. Find the constant of proportionality, k, in terms of the other variables, if the total mass of the pipe is M. (Recall that the total mass of a three-dimensional solid is given by the triple integral of the mass density over the region occupied by the solid.) Hint: Place the axis of the pipe along the z-axis and pick a coordinate system that simplifies the problem.

3. [2350/070921 Exam (18 pts)] Consider the solid shown below, which is a portion of the cone $x^2+y^2-\left(\frac{z}{3}\right)^2=0$.



Each of the following triple integrals can be used to compute the volume of $\mathcal{W}$. Copy each them onto your paper and provide the six (6) appropriate limits for each one, using the given order of integration. **Do not evaluate** any of the integrals. To receive full credit, you must use the correct bounds for the figure as shown (study it carefully), not bounds for an equivalent solid in a different octant.

(a) Volume ($\mathcal{W}$) = $\int\int\int \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$

(b) Volume ($\mathcal{W}$) = $\int\int\int r\,\mathrm{d}z\,\mathrm{d}r\,\mathrm{d}\theta$

(c) Volume ($\mathcal{W}$) = $\int\int\int \rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\theta\,\mathrm{d}\phi$

4. [2350/070921 Exam (22 pts)] Use polar coordinates to combine the following into a single double integral and then evaluate the resulting polar coordinate double integral. Making a sketch should prove beneficial.

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\,\mathrm{d}y\,\mathrm{d}x+\int_{1}^{\sqrt{2}}\int_{0}^{x} xy\,\mathrm{d}y\,\mathrm{d}x+\int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\,\mathrm{d}y\,\mathrm{d}x$$

5. [2350/070921 Exam (15 pts)] Evaluate $\displaystyle\int_0^4\int_0^1\int_{2y}^{2}\frac{2\cos(x^2)}{\sqrt{z}}\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z$. Hint: The antiderivative of $\cos(x^2)$ is not $-\sin\left(x^2\right)$.

6. [2350/070921 Exam (15 pts)] Using spherical coordinates with $\mathrm{d}V=\rho^2\sin\phi\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}\theta$, set up, but **do not evaluate**,

$$\iiint_Q \frac{z}{\sqrt{1+x^2+y^2}}\,\mathrm{d}V,$$

where Q is the region of the sphere $x^2+y^2+z^2=4$ below the plane $z=-\sqrt{3}$ and under the first octant.