1. [2350/062521 Exam (15 pts)] Consider the function $z = f(x, y) = 25x^2 + 4y^2 + 4$. On your paper write the letters (a)-(e) and next to each one write the word TRUE or FALSE as appropriate. No work is required and no partial credit will be given.

   (a) The level curves of $f(x, y)$ are circles.

   (b) The tangent plane to the function at $(x, y) = (0, 0)$ is horizontal.

   (c) The vertical trace of the function in the plane $y = 2$ is an ellipse.

   (d) If you were to walk along the curve $(2\cos t, 5\sin t)$ in the xy-plane, the height of the surface above you would be constant.

   (e) The domain and range of the function are, respectively, $\mathbb{R}^2$ and $[0, \infty)$.

2. [2350/062521 Exam (30 pts)] Parts (a) and (b) are not related.

   (a) (18 pts) A certain portion of a forest consists of its boundary, given by the lines $|x| = 5, |y| = 5$, and the region inside the boundary. The elevation in the forest is $h(x, y) = 3xy - x^3 - y^3 + 2$.

      i. (4 pts) A friend of yours (as well as your grader) does not want numbers or lengthy calculations, just a simple yes or no answer, with a verbal mathematical justification, to the question "Is there a highest and lowest point in the forest?"

      ii. (8 pts) Are there any saddles (passes) or local hills or valleys inside the boundaries of the forest? If so, find their locations and their elevations. If there are none, explain why not.

      iii. (6 pts) You and your friend are hiking along a trail whose projection onto the xy-plane is given by $(x(t), y(t)) = \left(t, \frac{1}{2}t^2\right)$.

         A. (2 pts) What is your elevation when $t = 2$?

         B. (4 pts) Use the chain rule to determine whether your elevation is increasing or decreasing when $t = 2$.

   (b) (12 pts) You are making a rectangular chicken run using 12 feet of fencing for a boundary. Use Lagrange Multipliers to find the dimensions of the run that will give the chickens the most area in which to frolic.

3. [2350/062521 Exam (15 pts)] Parts (a) and (b) are not related.

   (a) (6 pts) Find the following limits or show that they do not exist.

      i. $\displaystyle\lim_{(x,y,z)\to(\frac{\pi}{3},\frac{\pi}{6},\frac{\pi}{4})} \frac{\cos x + \sin y + \tan z}{\sqrt{3x + 6y + 4z}}$ ii. $\displaystyle\lim_{(x,y)\to(1,0)} \frac{1-x}{x+y-1}$

   (b) (9 pts) The mass of a certain object is given by the function $m = \frac{2}{3}\pi l^3 w^{3/2}$ where l is the length of the object and w is its width. Consider two such objects, one with a length of 1 unit and a width of 4 units and another with a length of 4 units and a width of 1 unit. Which object's mass is more sensitive to a small change in width? Justify your answer using mathematical techniques learned in this course.

4. [2350/062521 Exam (14 pts)] Jack and Jill went up a hill to the point (x_0, y_0, z_0) and got caught there in a lightning storm. In a fit of panic Jack darted off in the direction of $\mathbf{A} = 2\mathbf{i} - \mathbf{j}$ and noted at that instant that his altitude was changing at an instantaneous rate of $-\sqrt{5}/5$ ft/ft. Panic stricken as well, Jill ran in the direction of $\mathbf{B} = -3\mathbf{i} + \mathbf{j}$, noting an instantaneous rate of change of altitude of $-\sqrt{10}/5$ ft/ft. In what direction should they have run in order to start descending the hill at the fastest instantaneous rate? What would that rate have been?

5. [2350/062521 Exam (26 pts)] Parts (a) and (b) are not related.

   (a) (10 pts) Consider the function $g(x, y, z, t)$ where

   $$x = u^2 + v, \quad y = u + v^2, \quad z = \ln(v/u), \quad t = e^{uv}$$

   Suppose when $z = -\ln 4$ and $v = 1$ that $g_x = 2$, $g_y = -3$, $g_z = 6$ and $g_t = -2$. Calculate the instantaneous rate of change of $g(x, y, z, t)$ with respect to u at this point.

   (b) (16 pts) Consider the function $f(x, y) = \ln(xy)$.

      i. (8 pts) Calculate the first order Taylor (tangent plane) approximation (linearization) of f about the point $(1, 1)$.

      ii. (2 pts) Approximate the value of $f(1.1, 1.2)$ using your first order Taylor polynomial.

      iii. (6 pts) Find an upper bound on the error in your first order Taylor approximation over the region where $|x - 1| \le 0.1$ and $|y - 1| \le 0.2$.