

INSTRUCTIONS: Write your name and your instructor's name on the front of your work. Work all problems. Show your work clearly. Note that a correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Simplify all work to receive full credit.

1. (34 pts) The Calc 3 space cadets are back! During their travel back to Earth they accidentally travel counter-clockwise on the path of the left half of $x^2 + y^2 = 1$ connected by a straight line.
 - (a) (20 pts) Directly compute $\oint_C yx^2 dx - x^3 dy$.
 - (b) (14 pts) Use a Calc 3 theorem to compute part (a).
2. (34 pts) The space cadets have entered an asteroid field! To escape they'll need to calculate some line integrals. These parts are unrelated:
 - (a) (24 pts) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where \vec{F} is the conservative vector field $\langle 9x^2 - 3y^2x^2, 4 - 2yx^3 \rangle$ and C is the path parameterized as $\vec{r}(t) = \langle 3 - t^2, 5 - t \rangle$ with $t \in [-2, 3]$ using the Fundamental Theorem for Line Integrals.
 - (b) (10 pts) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is the ellipse $\frac{(x-5)^2}{4} + \frac{y^2}{9} = 1$ counter-clockwise if $\int_L \vec{F} \cdot d\vec{r}$ is path independent (L being any path). Be sure to motivate your answer. Note: There isn't a listed vector field \vec{F} on purpose.
3. (32 pts) The space cadet's ship is receiving the final plans for re-entry into Earth's atmosphere, but a few more calculations need to be done beforehand. These parts are unrelated:
 - (a) (16 pts) Given a function $f(x, y)$ with continuous second order partial derivatives and $x = 2u + uv$ and $y = v^2 - u$, what is $\frac{\partial^2 f}{\partial v^2}$ (you may have to leave certain expressions general)?
 - (b) (16 pts) Find the second order Taylor approximation of $f(x, y) = e^{-(x^2+y^2)}$ centered at the origin.
4. (20 pts) In preparation for the return to Earth's atmosphere the cadet's ship surrounds itself with a force field in the shape of the box $-1 \leq x \leq 2, 0 \leq y \leq 1$, and $1 \leq z \leq 4$. What is the flux of cosmic rays $\vec{F} = \langle \sin(\pi x), zy^3, z^2 + 4x \rangle$ out of the force field? Set up, but DO NOT EVALUATE, the calculation without directly computing any surface area integrals.
5. (30 pts) To finally land the spaceship the cadets need to use Stokes' Theorem to evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = \langle -yz, 4y + 1, xy \rangle$ and C is the circle of radius 3 perpendicular to the y-axis centered at $(0, 4, 0)$ with a clockwise rotation when looking down the y-axis from positive to negative.