

INSTRUCTIONS: Write your name and your instructor's name on the front of your work. Work all problems. Show your work clearly. Note that a correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit. Simplify all work to receive full credit.

- (25 pts) Dr. Strang (a real mathematician by the way) is helping by creating a math-mystical portal in the shape of a triangle with vertices $(0, 3)$, $(4, 1)$, and $(2, 6)$ in xy -space (let's call this region R). In order to complete the spell he needs to transform $\iint_R (x + 2y)dA$ using $x = \frac{1}{2}(u - v)$ and $y = \frac{1}{4}(3u + v + 12)$. Set up, but DO NOT EVALUATE, the resulting new integral in uv -space.
- (20 pts) Captain Mathematical is focusing her energy attacks in the area inside $r = 4 + 2\sin(\theta)$ and outside $r = 3 - \sin(\theta)$. Sketch the region and set up, but DO NOT EVALUATE, the integral that represents this area.
- (30 pts) Iron-math is flying around the battlefield. He spots groups of enemies sneaking behind two structures. In order to decide how much power to use in his attack he first needs data on these structures. Set up, but DO NOT EVALUATE the following integrals calculating:
 - The mass for the region bounded by $z = 0$, outside of $r = 1$, inside of $r = 1 + \cos(\theta)$, and $z = 3 - y$ with density $D(x, y, z) = x^2 + y^2 + z^2$ in the order $dzdrd\theta$.
 - The volume for the region bounded below by the xy -plane, on the sides by a sphere of radius 2 centered at the origin, and on the top by the cone $3z^2 = x^2 + y^2$ in the order $d\rho d\phi d\theta$
- (25 pts) Black Widom (Widom is also a real mathematician) is attempting to steal the villain's secret plans. She needs to break the computer's encryption first. In order to do so she needs to rewrite the integral

$$\int_{-1}^0 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{6x^2+6y^2}}^{\sqrt{7-x^2-y^2}} 18y dz dy dx$$

in a different coordinate system in order to evaluate it. This time set up AND evaluate this integral. **Hint:** If you are struggling evaluating the integral maybe try a different integration order.