INSTRUCTIONS: Write your name and your instructor's name on the front of your work. Work all problems. Show your work clearly. Note that a correct answer with incorrect or no supporting work may receive no credit, while an incorrect answer with relevant work may receive partial credit.

1. (20 points) Prince Leo has begun a math-mystical journey across the realm. He comes to long stone bridge over a deadly canyon. A strange man in grey clothing yells across "You shall not pass... without solving these riddles". The man asks to consider a function of two variable, $f(x, y)$, that has continuous second order derivatives. He states that $x = u^2 + 3v$ and $y = uv$ and asks Leo to find the following (you may have to leave certain expressions general):

   (a) $\frac{\partial f}{\partial u}$ (b) $\frac{\partial}{\partial u}\frac{\partial f}{\partial x}$ (c) $\frac{\partial}{\partial u}\frac{\partial f}{\partial y}$ (d) $\frac{\partial^2 f}{\partial u^2}$

2. (30 points) After crossing the bridge Leo enters a large cavern. Inside he finds magic light swirling around a large sphere (centered at the origin) of radius 6m. Leo finds that the amount of light can be described by $f(x, y, z) = y^2 - 10z$ (allowing magic light to have negative values). What is the maximum and minimum values of magic light on the surface of the sphere?

3. (30 points) Leo exits the magic cavern near the top of Mount Newton. Locally, The elevation is described by $h(x, y) = 7x - 8y + 2xy - x^2 + y^3$.

   (a) Find and classify all the critical points of Mount Newton.

   (b) Leo is currently at $(1, -1, 11)$ when a mighty dragon descends from the sky breathing fire. Leo quickly runs down the mountain in the steepest direction. Find a full 3-dimensional vector that points in the direction Leo begins running.

4. (20 points) Leo continues running from the dragon until he encounters a sudden cliff. With nowhere to run he turns to fight the beast. Surprisingly, the dragon challenges Leo to a math duel. The dragons roars to

   (a) Find the second order Taylor approximation to $e^x \sin(y)$ at the origin.

   (b) Estimate the error in this approximation if $|x| \le 0.1$ and $|y| \le 0.1$

   After Leo returns from his journey he is finally ready to take the throne to become King Leonhard Euler.