Write on the front of your bluebook a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 4 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You are taking this exam in a proctored and honor code enforced environment. Thus, no notes/papers, calculators, cell phones, or other electronic devices are permitted.

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1. [30 pts] The following questions are not related.
   (a) [6 pts] Let $F(x, y, z) = \left(1 - x^2\right)^{1/2} e^{y^2 + z^{-1/4}}$.
      i. [4 pts] State the domain and range of $F(x, y, z)$.
      ii. [2 pts] Describe the level surfaces $F(x, y, z) = 0$.
   (b) [5 pts] Let $h(x, y) = \dfrac{x + 2y}{3x^2 - 12y^2}$.
      i. [3 pts] Find $\lim\limits_{(x,y)\to(4,-2)} h(x, y)$ or explain why the limit does not exist.
      ii. [2 pts] Is $h(x, y)$ continuous at $(4, -2)$? Explain briefly.
   (c) [9 pts] Use the chain rule to find $\partial w/\partial v$ at the point $(u, v) = (-1, 2)$ if $w = xy + \ln z$, $x = u^2/v$, $y = e^{u+3v}$ and $z = \cos^2(\frac{\pi}{8} v)$.
   (d) [10 pts] The magnetic field in a portion of space is given by $M = \ln(xyz)$. To recharge your spaceship's fuel cells in the most efficient way possible, you want to guide your ship in the direction that produces the greatest rate of change of the magnetic field with respect to distance. When you are at the point $(1, 1, 2)$, find the direction you should aim your ship and determine the corresponding rate of change of the magnetic field.
2. [20 pts] Let $T(x, y) = \cos(\pi xy) + xy^2$ give the temperature at a point (x, y) where the positive x–axis points to the east and the positive y–axis points to the north.
   (a) [10 pts] You and a friend begin walking from the point $\left(\frac{1}{4}, 2\right)$. You walk straight to the east and your friend walks straight to the north, both walking the same short distance. Using differentials, estimate which of you experiences the greater temperature change.
   (b) [10 pts] Another friend of yours is walking along the path given by $\mathbf{r}(t) = \dfrac{1}{\sqrt{t}}\,\mathbf{i} + \dfrac{t^2}{16}\,\mathbf{j}$, $t > 0$. Is the temperature increasing or decreasing with respect to time as this friend passes through the point $\left(\frac{1}{2}, 1\right)$? At what rate?
3. [20 pts] A sweet new amusement park ride lets you wander around on the surface of the cone $z^2 = x^2 + y^2$. A friend of yours is standing at the point $P(3, 4, 0)$. Using Lagrange multipliers, determine the closest you will get to your friend. Where will you be when this occurs?
4. [30 pts] You are to locate potential sites for forest ranger patrol cabins in a section of forest where the elevation of the ground is described by the function $f(x, y) = xy - \dfrac{x^3}{3} - \dfrac{y^3}{3}$. The main requirement is that the ground be level where the cabins are to be located.
   (a) [10 pts] Determine the location of all potential cabin sites.
   (b) [10 pts] Classify these potential cabin sites (hill top, valley bottom, *etc.*) and determine the elevation of each of these locations.
   (c) [10 pts] If you were to build a network of roads directly connecting each cabin site to another, what would be the steepest slope you would encounter when driving along these roads? Hint: Think carefully about what you are trying to maximize here.

FORMULAS ON BACK

PROJECTIONS, DISTANCE FROM POINT S TO LINE PARALLEL TO $\mathbf{v}$ CONTAINING POINT P; DISTANCE FROM POINT S TO PLANE WITH NORMAL $\mathbf{n}$ CONTAINING POINT P

$$\operatorname{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a} \qquad d = \frac{\left\|\overrightarrow{PS}\times\mathbf{v}\right\|}{\|\mathbf{v}\|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{\|\mathbf{n}\|}\right|$$

ARC LENGTH, FRENET FORMULAS, AND TANGENTIAL AND NORMAL ACCELERATION COMPONENTS

$$\mathrm{d}s = \|\mathbf{v}\|\,\mathrm{d}t \qquad \mathbf{T} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = \frac{\mathbf{v}}{\|\mathbf{v}\|} \qquad \mathbf{N} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}s}{\|\mathrm{d}\mathbf{T}/\mathrm{d}s\|} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}t}{\|\mathrm{d}\mathbf{T}/\mathrm{d}t\|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s} = \kappa\mathbf{N} \quad \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s} = -\tau\mathbf{N} \quad \kappa = \left\|\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s}\right\| = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|^3} = \frac{|f''(x)|}{\left\{1+[f'(x)]^2\right\}^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}} \quad \tau = -\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s}\cdot\mathbf{N}$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N} \qquad a_T = \frac{\mathrm{d}\|\mathbf{v}\|}{\mathrm{d}t} \qquad a_N = \kappa\|\mathbf{v}\|^2 = \sqrt{\|\mathbf{a}\|^2 - a_T^2}$$

DIRECTIONAL DERIVATIVE, DISCRIMINANT, AND LAGRANGE MULTIPLIERS

$$D_{\mathbf{u}}f = \frac{\mathrm{d}f}{\mathrm{d}s} = (\nabla f)\cdot\mathbf{u} \qquad D = f_{xx}f_{yy} - (f_{xy})^2 \qquad \nabla f = \lambda\nabla g, \quad g = 0$$