Write on the front of your bluebook a grading key, your name, student ID, your lecture number and instructor. This exam is worth 100 points and has 4 questions.

- Submit this exam sheet with your bluebook. However, nothing on this exam sheet will be graded. Make sure all of your work is in your bluebook.
- **Show all work and simplify your answers!** Answers with no justification will receive no points unless otherwise noted. **Please begin each problem on a new page.**
- You are taking this exam in a proctored and honor code enforced environment. Thus, no notes/papers, calculators, cell phones, or other electronic devices are permitted.

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1. [24 pts] The following problems are not related.

   (a) Plane P_1 is given by $x - y + z = 1$ and plane P_2 is given by $x + y + z = 1$. Find the projection of the normal vector of P_1 onto the normal vector of P_2.

   (b) You are playing around with your new laser pointer inside your house. Part of the ceiling can be described by $x - y + 4z = 24$. If the beam from the laser pointer travels along the line whose symmetric equations are

   $$x - 1 = \frac{y+1}{3} = \frac{z-2}{4},$$

   will the beam hit the ceiling? If so, find the coordinates of the point where the beam shines on the ceiling. If not, explain why not.

   (c) Find the equation of the plane containing the points $P_1(-2, 1, 4)$ and $P_2(1, 0, 3)$ that is perpendicular to the plane $4x - y + 3z = 2$. Write your answer in the form $ax + by + cz = d$.

2. [24 pts] The following problems are related.

   (a) Find an equation of the surface consisting of all points that are equidistant from the point $(1, -2, 0)$ and the plane $y = -4$, simplifying your answer.

   (b) Identify the surface.

   (c) Find the equation of the curve of intersection of the surface and the yz–plane. Simplify your answer.

3. [40 pts] A satellite orbits a planet (whose center is located at the origin) along the trajectory (path)

   $$\mathbf{r}(t) = \cos(\pi t)\ \mathbf{i} + \sqrt{3}\sin(\pi t)\ \mathbf{j} + \sqrt{2}\cos(\pi t)\ \mathbf{k}$$

   where t is in hours and x, y, z are in kilometers.

   (a) Calculate the speed of the satellite.

   (b) Calculate $\mathbf{T}(t)$ and $\mathbf{N}(t)$ for the satellite's trajectory.

   (c) Calculate the curvature of the satellite's path.

   (d) What are the tangential and normal components of the acceleration of the satellite?

   (e) At time $t = 3$ hours, the satellite detects an asteroid straight ahead at a distance of 4 km. Find the coordinates of the asteroid.

4. [12 pts] In your blue book, write **TRUE** if the statement is true or write **FALSE** if the statement is false. No justification required and no partial credit given.

   (a) $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

   (b) The curvature of a path is a measure of how the unit tangent vector changes as one moves along the path.

   (c) If $\mathbf{u}$ and $\mathbf{v}$ are three dimensional vectors, then $|\mathbf{u} \cdot \mathbf{v}| \le \|\mathbf{u}\|\|\mathbf{v}\|$

   (d) If a particle moves along a smooth path in the plane $x = -7$, then its unit binormal vector $\mathbf{B}$ has only a $\mathbf{j}$–component.

   (e) If $\mathbf{u}$ and $\mathbf{v}$ are two nonzero vectors such that $\mathbf{u} \times \mathbf{v} = \mathbf{0}$, then there exists a real number c such that $\mathbf{u} = c\mathbf{v}$.

   (f) If $\dfrac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = -t\,\mathbf{i} - t\,\mathbf{j} - t\,\mathbf{k}$ and $\mathbf{r}(1) = \mathbf{i} + 2\,\mathbf{j} + 3\,\mathbf{k}$, then $\mathbf{r}(t) = \frac{1}{2}\left[(3 - t^2)\mathbf{i} + (5 - t^2)\mathbf{j} + (7 - t^2)\mathbf{k}\right]$

FORMULAS ON BACK

PROJECTION; DISTANCE FROM POINT S TO LINE PARALLEL TO $\mathbf{v}$ CONTAINING POINT P; DISTANCE FROM POINT S TO PLANE WITH NORMAL $\mathbf{n}$ CONTAINING POINT P

$$\operatorname{proj}_{\mathbf{a}}\mathbf{b} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{a}\cdot\mathbf{a}}\right)\mathbf{a} \qquad d = \frac{\left\|\overrightarrow{PS}\times\mathbf{v}\right\|}{\|\mathbf{v}\|} \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{\|\mathbf{n}\|}\right|$$

ARC LENGTH, FRENET FORMULAS, AND TANGENTIAL AND NORMAL ACCELERATION COMPONENTS

$$\mathrm{d}s = \|\mathbf{v}\|\,\mathrm{d}t \qquad \mathbf{T} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}s} = \frac{\mathbf{v}}{\|\mathbf{v}\|} \qquad \mathbf{N} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}s}{\|\mathrm{d}\mathbf{T}/\mathrm{d}s\|} = \frac{\mathrm{d}\mathbf{T}/\mathrm{d}t}{\|\mathrm{d}\mathbf{T}/\mathrm{d}t\|} \qquad \mathbf{B} = \mathbf{T}\times\mathbf{N}$$

$$\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s} = \kappa\mathbf{N} \quad \frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s} = -\tau\mathbf{N} \quad \kappa = \left\|\frac{\mathrm{d}\mathbf{T}}{\mathrm{d}s}\right\| = \frac{\|\mathbf{v}\times\mathbf{a}\|}{\|\mathbf{v}\|^3} = \frac{|f''(x)|}{\left\{1+[f'(x)]^2\right\}^{3/2}} = \frac{|\dot{x}\ddot{y}-\dot{y}\ddot{x}|}{(\dot{x}^2+\dot{y}^2)^{3/2}} \quad \tau = -\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}s}\cdot\mathbf{N}$$

$$\mathbf{a} = a_T\mathbf{T} + a_N\mathbf{N} \qquad a_T = \frac{\mathrm{d}\|\mathbf{v}\|}{\mathrm{d}t} \qquad a_N = \kappa\|\mathbf{v}\|^2 = \sqrt{\|\mathbf{a}\|^2 - a_T^2}$$