APPM 2350, Summer 2018: Exam 3 July 13, 2018

INSTRUCTIONS: Please show all of your work and make your methods and reasoning clear. Answers out of the blue with no supporting work will receive no credit (unless the directions to a specific problem say otherwise, of course). No calculators or electronic devices are allowed. Please start each numbered problem on a new page in your bluebook and do the problems in order. Please sign the front of your blue book indicating you read and understood these directions in addition to the CU honor code.

SIMPLIFY ALL ANSWERS AS FAR AS POSSIBLE!!!

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1. (20 points) The integral

$$\int_0^4 \int_{\sqrt{y}}^2 1\, dx\, dy$$

represents the area of a thin, flat plate R in the xy-plane.

(a) Sketch a graph of the region R, labeling anything of interest including the axes.

(b) Find the mass of the plate given the density function $\rho(x, y) = \sin(x^3)$.

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2. (24 points) Consider the rectangular region R in the first quadrant in the xy-plane bounded by $x = 0$, $y = 0$, $x = a$ and $y = b$ where a and b are positive constants with $a < b$. Please set up the double integral(s) in polar coordinates in the order $d\theta\, dr$ that equal the area of R. You don't have to actually do the integral(s), just set them up! (The area is ab of course, but that's not the question!) Include a relevant sketch as part of your answer.

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3. (24 points) Evaluate the integral

$$\iint_R \frac{x - 2y}{3x - y}\, dA$$

where R is the parallelogram enclosed by the lines $x = 2y$, $x = 2y + 4$, $y = 3x - 1$ and $y = 3x - 8$. Include any relevant sketches in your solution.

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4. (32 points) The integrals

$$V = \int_0^{2\pi} \int_0^{\pi/3} \int_0^{2\cos(\phi)} \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta + \int_0^{2\pi} \int_{\pi/3}^{\pi/2} \int_0^1 \rho^2 \sin(\phi)\, d\rho\, d\phi\, d\theta$$

determine the volume of a solid object.

(a) Make a clear sketch of the cross section of the object in the rz-plane (this is a constant θ plane in cylindrical coordinates).

(b) What does the solid look like? Try to describe it in a few words of your own, technical or not, it doesn't matter. Or you can sketch a graph if you want. You shouldn't spend a lot of time on this.

(c) Express V in spherical coordinates using the order $d\phi\, d\rho\, d\theta$.

(d) Express V in cylindrical coordinates using the order $dz\, dr\, d\theta$.

(e) Evaluate any of the integrals above (including the original) to determine the value of V.

Projection, distances from a point S to a line containing a point P, and S to a plane with normal $\mathbf{n}$:

$$\text{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b}, \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|}, \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}, \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}, \qquad \mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}, \qquad \kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}, \qquad \kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}, \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta), \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\frac{df}{ds} = \nabla f\cdot\mathbf{u} \qquad D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - (f_{xy}(x,y))^2 \qquad \nabla f = \lambda\nabla g,\ g = 0$$

Third order Taylor approximation of $f(x,y)$ centered at (a,b):

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b) + \frac{1}{2!}\left[f_{xx}(a,b)(x-a)^2 + 2f_{xy}(a,b)(x-a)(y-b) + f_{yy}(y-b)^2\right]$$
$$+ \frac{1}{3!}\left[f_{xxx}(a,b)(x-a)^3 + 3f_{xxy}(a,b)(x-a)^2(y-b) + 3f_{xyy}(a,b)(x-a)(y-b)^2 + f_{yyy}(a,b)(y-b)^3\right]$$

n^{th} order Taylor approximation error centered at the point (a,b):

$$|E(x,y)| \leq \frac{M}{(n+1)!}\left(|x-a| + |y-b|\right)^{n+1}$$

where M is an upper bound on the absolute value of all the relevant $n+1^{st}$ order partial derivatives of f.