APPM 2350, Summer 2018: Exam 2 June 29, 2018

INSTRUCTIONS: Please show all of your work and make your methods and reasoning clear. Answers out of the blue with no supporting work will receive no credit (unless the directions to a specific problem say otherwise, of course). No calculators or electronic devices are allowed. Please start each numbered problem on a new page in your bluebook and do the problems in order. Please sign the front of your blue book indicating you read and understood these directions in addition to the CU honor code.

**SIMPLIFY ALL ANSWERS AS FAR AS POSSIBLE!!!**

---

1. (24 points) A few unrelated questions. Show your work in this problem.

(a) Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$ or explain why it doesn't exist.

(b) Find a vector equation of the tangent line at the point $(-2, 2, 4)$ to the curve of intersection of the surface $z = 2x^2 - y^2$ and the plane $z = 4$.

---

2. (24 points) A few unrelated questions. You don't need to show any work in this problem and no partial credit will be given.

(a) TRUE or FALSE? If $f(x, y) = \cos(x) + \cos(y)$, then $-\sqrt{2} \leq D_{\mathbf{u}} f(x, y) \leq \sqrt{2}$ for any unit vector $\mathbf{u}$ and any point $(x, y) \in \mathbb{R}^2$.

(b) Suppose $z = f(x, y)$ where $x = g(s, t)$, $y = h(s, t)$ and you are given:

$$\begin{array}{lll} f(3,6)=0 & f_x(3,6)=7 & f_y(3,6)=8 \\ g(1,2)=3 & g_s(1,2)=-1 & g_t(1,2)=4 \\ h(1,2)=6 & h_s(1,2)=-5 & h_t(1,2)=10 \end{array}$$

Find $\dfrac{\partial z}{\partial s}$ and $\dfrac{\partial z}{\partial t}$ when $s = 1$ and $t = 2$.

(c) Find the point(s) on the surface $x^2+4y^2-z^2 = 4$ where the tangent plane is parallel to $2x+2y+z = 5$.

---

3. (22 points) **Use Lagrange multipliers** to find the absolute maximum and minimum values of $f(x, y, z) = 2x + 2y + z$ subject to the constraint $x^2 + y^2 + z^2 = 9$. Show your work in this problem.

---

**OVER FOR MORE PROBLEMS!**

4. (30 points) Consider the function $f(x, y) = \sin(x)\sin(y)$. Show your work in this problem.

(a) Find the critical point(s) of $f$ that satisfy $-\pi < x < \pi$ and $-\pi < y < \pi$.

(b) Classify the critical point(s) you found in part (a) as the location of local maxima, local minima, or saddle points.

(c) At the point $(\pi/4, \pi/4)$, in what direction does $f$ increase most rapidly?

(d) What is the maximum rate of change of $f$ with respect to distance at the point $(\pi/4, \pi/4)$?

(e) Find the linear approximation of $f(x, y)$ at the point $(\pi/4, \pi/4)$.

(f) Use your answer from part (e) to approximate $f(\pi/4 + 0.1, \pi/4 + 0.1)$.

(g) Now, calculate an upper bound on the error associated with the linear approximation you found in part (e) that is valid for *any* values of $x$ and $y$ that satisfy $|x - \pi/4| \leq 0.25$ and $|y - \pi/4| \leq 0.5$.

---

Projection, distances from a point $S$ to a line containing a point $P$, and $S$ to a plane with normal $\mathbf{n}$:

$$\text{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b}, \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|}, \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}, \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}, \qquad \mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}, \qquad \kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}, \qquad \kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}, \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta), \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$

$$\frac{df}{ds} = \nabla f\cdot\mathbf{u} \qquad D(x,y) = f_{xx}(x,y)f_{yy}(x,y) - (f_{xy}(x,y))^2 \qquad \nabla f = \lambda\nabla g,\ g = 0$$

Third order Taylor approximation of $f(x, y)$ centered at $(a, b)$:

$$f(x,y) \approx f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b) + \frac{1}{2!}\left[f_{xx}(a,b)(x-a)^2 + 2f_{xy}(a,b)(x-a)(y-b) + f_{yy}(y-b)^2\right]$$
$$+ \frac{1}{3!}\left[f_{xxx}(a,b)(x-a)^3 + 3f_{xxy}(a,b)(x-a)^2(y-b) + 3f_{xyy}(a,b)(x-a)(y-b)^2 + f_{yyy}(a,b)(y-b)^3\right]$$

$n^{th}$ order Taylor approximation error centered at the point $(a, b)$:

$$|E(x,y)| \leq \frac{M}{(n+1)!}\left(|x-a| + |y-b|\right)^{n+1}$$

where $M$ is an upper bound on the absolute value of all the relevant $n + 1^{st}$ order partial derivatives of $f$.