APPM 2350, Summer 2018: Exam 1 June 15, 2018

INSTRUCTIONS: Please show all of your work and make your methods and reasoning clear. Answers out of the blue with no supporting work will receive no credit (unless the directions to a specific problem say otherwise, of course). No calculators or electronic devices are allowed. Please start each numbered problem on a new page in your bluebook and do the problems in order. Please sign the front of your blue book indicating you read and understood these directions in addition to the CU honor code.

SIMPLIFY ALL ANSWERS AS FAR AS YOU CAN

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1. (25 points) Consider the plane $x + y + 2z = 1$.

(a) Sketch a graph of the part of the plane that is in the first octant. Label your axes and any points of interest.

(b) Using only Calculus 3 methods, find the area of the shape you graphed in part (a).

(c) Find the point on the plane that is closest to the origin and say what this minimum distance is **from scratch using projections. Don't just plug stuff into a formula, show your reasoning!**.

(d) Find an equation of the line through the origin and perpendicular to the plane.

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2. (24 points) A few unrelated questions. Just provide answers for this problem – you don't have to show any work and no partial credit will be given.

(a) TRUE or FALSE? If $\mathbf{a} + \mathbf{b}$ and $\mathbf{a} - \mathbf{b}$ are orthogonal, then the vectors $\mathbf{a}$ and $\mathbf{b}$ must have the same length.

(b) Classify the quadric surface $x^2 + y^2 + z^2 - 2x - 4y - 8z = 15$ as completely as you can.

(c) Consider the surfaces $z = x^2$ and $x^2 + y^2 = 1$.

(i) Describe both surfaces using two words each.

(ii) Find a vector-valued function that represents the curve of intersection of the two surfaces using basic trig functions.

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3. (25 points) Consider the function $\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t), e^t \rangle$ on the interval $0 \le t \le \pi$.

(a) Describe the given curve in one sentence as specifically as you can.

(b) Find the point on the curve where the tangent line is parallel to the plane $\sqrt{3}x + y = 1$.

(c) Set up a simplified integral that represents the arc length of the given curve. **You do not have to actually evaluate this integral!**

OVER FOR MORE PROBLEMS!

4. (26 points) Consider a particle with position function $\mathbf{r}(t) = \langle t\cos(t), t\sin(t), t\rangle$.

(a) Find the velocity of the particle when $t = 0$.

(b) Find the speed versus time function $v(t)$.

(c) Find the speed of the particle when $t = 0$.

(d) Find the unit tangent vector $\mathbf{T}(0)$.

(e) Find the acceleration vector $\mathbf{a}(0)$.

(f) Find the curvature κ when $t = 0$.

(g) Now, I assume you don't want to calculate $\mathbf{N}(0)$ directly from its definition. Don't worry, you don't have to do that, so please don't. However, I wonder if you could find $\mathbf{N}(0)$ indirectly using what you've already done in parts (a) through (f) of this problem and a little bit of work. What do you think? Could you? Simply answer YES or NO.

Projection, distances from a point S to a line containing a point P, and S to a plane with normal $\mathbf{n}$:

$$\text{proj}_{\mathbf{b}}\mathbf{a} = \left(\frac{\mathbf{a}\cdot\mathbf{b}}{\mathbf{b}\cdot\mathbf{b}}\right)\mathbf{b}, \qquad d = \frac{|\overrightarrow{PS}\times\mathbf{v}|}{|\mathbf{v}|}, \qquad d = \left|\overrightarrow{PS}\cdot\frac{\mathbf{n}}{|\mathbf{n}|}\right|$$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}, \qquad \mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}, \qquad \mathbf{B}(t) = \mathbf{T}(t)\times\mathbf{N}(t)$$

$$\frac{d\mathbf{T}}{ds} = \kappa\mathbf{N}, \qquad \kappa(t) = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}, \qquad \kappa(x) = \frac{|f''(x)|}{[1+(f'(x))^2]^{3/2}}, \qquad \frac{d\mathbf{B}}{ds} = -\tau\mathbf{N}$$

$$\sin(2\theta) = 2\sin(\theta)\cos(\theta), \qquad \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$$